Proposed solutions for tutorials 1&2

Intermediate Microeconomics (UTS 23567)[*]

Preliminary and incomplete

Available at https://backwardinduction.blog/tutoring/

Office hours on Mondays from 9 am till 10 am in building 8 on level 9

Please whatsapp me on 0457871540 so I could meet you at the door, I don't have an internal phone

Also please whatsapp if you have questions, I won't be able to answer through whatsapp but I will give answer during office hours or in the beginning of next tutorial

Sergey V. Alexeev

2018

Navigation

[*] Questions for the tutorial were provided by Massimo Scotti, slides (I think) and textbook are by Nechyba (2016). Solutions and commentary by Sergey Alexeev (e-mail: sergei.v.alexeev@gmail.com).

Tutorial 1 (20/03/2018)

A prequel to this story is chapter 2 of Nechyba (2016). However, chapter 3 provides good examples for stuff from chapter 2, which I'd say is more interesting.

I will start with a general remark which is a bit different from what you heard in class or read in the textbook. The remark is meant to give a compact and intuitive understand and intends to complement the outlined subject narrative. I hope it makes sense.

Imagine that a manager at the company wants to pick the most productive guy for a job, or a girl that tries to pick the most devoted admirer for a boyfriend, or even a member of an admission committee that has to pick the ablest college applicants. An ideal solution is to rank all candidates along an important for particular context characteristic and pick the best one. Then the most hardworking gets a job, the most loving gets a girlfriend and the smartest goes to college. The problem, however, is that such a ranking requires us to see what can't be seen. The true level of productivity, devotion or ableness are unobserved. Asking candidates won't work, all of them have an incentive to lie to be selected.

The solution, however, can be found by recognizing that the most productive is likely to have the highest educational degree, the most devoted is likely to bring the biggest present on Valentine's Day, and the smartest is likely to have highest exam marks.

These relationships are not accidental. All three were intentionally created as a cultural truth-telling solution in the contexts where ksnowing the observable is important but candidates lack an incentive to reveal them truthfully.

In fact, the efficiency of a market economy is built on this truth-telling property. An economic system achieves maximum efficiency if those who value resources the most also possess them. Simply asking won't do due to misaligned incentives, that is why we ask everyone to name a price they comfortable to pay. A willingness to pay contains information on the importance of a good to an economics agent. That is how a resource gets an efficient allocation.

Most of economics is built around connecting an important unobservable and related to it observable that allows to make better choices. What we observe contain information on something important that we do not observe and those two are related according to some rule. The better we understand that rule the more information can be retrieved about that unobservable and the better the choices are.

$$\underbrace{\{1,2,3,...\}}_{\text{Unobservable}} \xrightarrow{rule} \underbrace{\{1,2,3...\}}_{\text{Observable}}$$

Fortunate for economics mathematicians developed a notion of a mathematical function. Which is precisely the idea that two sets are related according to some rule.

Microeconomics starts on a premise that observed *choices* are not senseless random act and if properly studied can reveal information on unobserved *tastes*. That is *choices* are actually functions $choices = f(tastes)$ that take *tastes* and map them into *choices* according to some rule. Or, put differently, *choices* are manifestation of *tastes*. To learn as much as possible from *choices* economists has to focus on a mapping $f(\cdot)$ that connect those two. This mapping can be interpreted as a rational behaviour that for given *tastes* produces *choices*.

This tutorial discuss one aspect of this mapping. In particular that decisions are made within a choice set.

A motivating example. We know that Americans buy a lot of guns and very little Vegemite, while Australians buy a lot of Vegemite and don't buy guns. A natural thing to conclude is that Americans really like guns and Australian really like Vegemite. Not necessarily. Maybe Australians like guns no less then Americans. It can't be infer it from the *choices* because guns are not in a choice set of a typical Australian household. However, Vegemite is in a choice set, and we can reason that Australian do indeed like it and much more than Americans. Let's make this intuition a little more precise.

Question 1

Consider the following expression:

$$x_a p_a + x_b p_b = m \tag{1}$$

where

- x_a and x_b denote the quantities of good a and b respectively;
- p_a and p_b denote the prices of goods a and b respectively;
- m denotes the income of a consumer.

1.1. What does equation (1) represent?

Answer to 1.1

If m is all the money you have for goods a and b then buying more of good a can only happen if you buy less of good b. Formally it means that this equation should be satisfied:

$$x_a p_a + x_b p_b = m$$
$$\Updownarrow$$
$$\underbrace{\begin{array}{c}\text{(Price of one unit of good } a\text{)} \\ \times \text{ (I buy } x_a \text{ units of good } a\text{)}\end{array}}_{\text{Total spending on good } a} + \underbrace{\begin{array}{c}\text{(Price of one unit of good } b\text{)} \\ \times \text{ (I buy } x_b \text{ units of good } b\text{)}\end{array}}_{\text{Total spending on good } b} = \text{All I had to spend on } a \text{ and } b.$$

Here x_a and x_b are choices, something a person has a control over. However, p_a and p_b and m are given and the decision maker can't do anything about it.

Of course he/she can always wait a little longer to save more money (increase m) or try a vendor with different prices (decrease p_a, p_b) but even if he/she does it there still exists a short enough period of time when those considered as fixed and only the amount of purchased x_a and x_b can be changed. In other words, there is always exist a (potentially very) short enough period of time when this budget line exists and needs to be respected.

It sort of like we know that the Earth is round, but to study the durability of a building we treat it as flat.

Question 1

Consider the following expression:

$$x_a p_a + x_b p_b = m \tag{1}$$

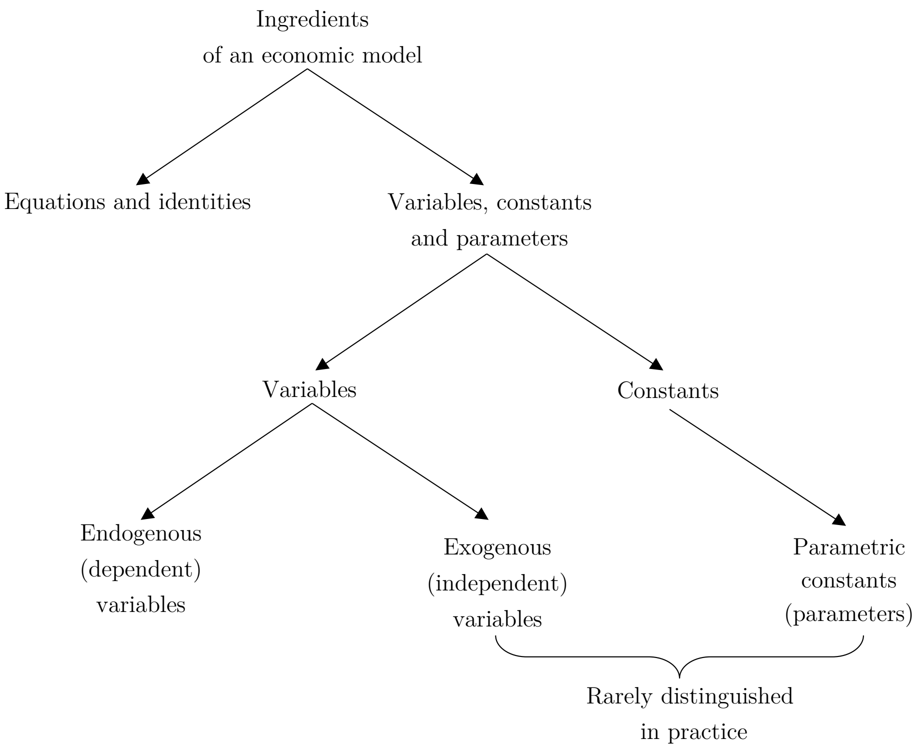
where

- x_a and x_b denote the quantities of good a and b respectively;
- p_a and p_b denote the prices of goods a and b respectively;
- m denotes the income of a consumer.

1.2. Equation (1) contains 5 letter. Which letters do we treat as "variables"? Which ones as "constants" or "parameters"? Why?

Answer to 1.2

See ingredients of an economics model by Wainwright et al. (2005) for the details.



The bottom line, however, is to distinguish parameters (aka constants, exogenous variables) and endogenous variables (aka variables). The exact differences are technical and very rarely paid attention to.

To understand where terminology comes from let us consider some function

$$x_b = f(x_a; m, p_b, p_a)$$

that take a point x_a from some set $X_a = \{x_{a_1}, x_{a_2}, x_{a_3}, ...x_{a_n}\}$ and associate it with another point x_b in some other set $X_b = \{x_{b_1}, x_{b_2}, x_{b_3}, ...x_{b_n}\}$.

To see better that the function is a particular relation between two sets sometimes functions are written in this form:

$$f : X_a \to Y_b.$$

Either way it should be read as x_b is a point that is associated with x_a and the rule of association is $f(\cdot)$. Parameters belong to the mapping, not to the sets. And this is the key difference.

A letter before semicolon is a variable, letters after are parameters.

$$\underbrace{x_b}_{\substack{\text{endogenous}\\\text{variable}}} = f(\underbrace{x_a}_{\substack{\text{endogenous}\\\text{variable}}} ; \underbrace{m, p_b, p_a}_{\text{parameters.}})$$

Variables represent something that we want to study and usually we do it by assuming some parametric relationship between variables of interest.

Sometimes we assume some particular values for the parameters, but we never assume particular values for variables, we always want them to be solved endogenously in the model as a dependent variable.

Here is one example

$$x_b \stackrel{def}{=} f(x_a; m, p_b, p_a) = \frac{m}{p_b} - \frac{p_a}{p_b} x_a.$$

It says that x_b is defined as a function. Then the function takes on an exact (parametric) form, in other words, we have related variables x_a and x_b through a rule and the rules is characterized by parameters. That is why choosing a function also sometimes called a parametrization of a relationship.

Finally also note that once a parametric mapping $f(\cdot)$ is defined parameters and variables can be chosen arbitrary. For example each of these four define a particular x_b

$$\begin{aligned} f(x_a; m, q, p_a) &= \frac{m}{p_b} - \frac{p_a}{q} x_a \\ f(x_a; 2\gamma, p_b, p_a) &= \frac{2\gamma}{p_b} - \frac{p_a}{p_b} x_a \\ f(10; m, p_b, p_a) &= \frac{m}{p_b} - \frac{p_a}{p_b} 10 \\ f(10; m, p_b, w) &= \frac{m}{p_b} - \frac{w}{p_b} 10. \end{aligned}$$

Meaning we might as well do the following:

$$f(x_a; b^2, b, kp_b) = b^2 \frac{1}{b} - kp_b \frac{1}{p_b} x_a = b - kx_a = f(x_a; b, k)$$

which says that by picking parameter we can bring the function to a linear form with intercept b and a slope $-k$.

Question 1

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where

- x_a and x_b denote the quantities of good a and b respectively;
- p_a and p_b denote the prices of goods a and b respectively;
- m denotes the income of a consumer.

1.3.

Plot equation (1) in graph where you measure x_b on the vertical axis and x_a on the horizontal axis.

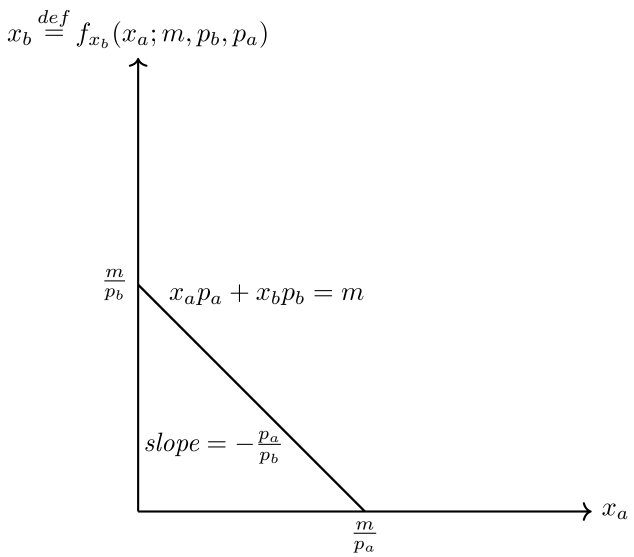
1.3.i

Identify the expression of the slope, vertical intercept and horizontal intercept. What does the slope represent?

1.3.ii

Show graphically what happens if the government provides the consumer with a subsidy that doubles the income of the consumer

Answer to 1.3



Answer to 1.3i

The budget line (1) can be rearranged in terms of variable x_a and parameters m, p_b, p_a

$$\begin{aligned} p_a x_a + p_b x_b &= m \\ p_b x_b &= m - p_a x_a \\ x_b &= \frac{m}{p_b} - \frac{p_a}{p_b} x_a. \end{aligned} \tag{NB}$$

Then x_b (the amount of good b) can be understood as a function of x_a (the amount of good a) and some parameters m, p_b, p_a. The following defines x_b as a function of a variable x_a and parameters m, p_b, p_a (a little subscript $_{x_b}$ just to remind us that even though it is a function it is also x_b)

$$x_b \stackrel{def}{=} f_{x_b}(x_a; m, p_b, p_a).$$

Because we already know the exact parametric relationship we write

$$f_{x_b}(x_a; m, p_b, p_a) = \underbrace{\frac{m}{p_b}}_{\substack{\text{vertical}\\ \text{intercept}}} - \underbrace{\frac{p_a}{p_b}}_{\text{slope}} x_a.$$

This function then assigns (governs) how much of good b can be bought if a particular amount of good a has already been bought given values of parameters m, p_b, p_a.

From the figure it is clear that the horizontal intercept is a point where $x_b \overset{def}{=} f_{x_b}(x_a; m, p_b, p_a)$ achieves zero:

$$\begin{aligned}
f_{x_b}(x_a; m, p_b, p_a) &\overset{set}{=} 0\\
\frac{m}{p_b} - \frac{p_a}{p_b}x_a &= 0\\
\frac{m}{p_b} &= \frac{p_a}{p_b}x_a\\
m &= p_a x_a\\
x_a &= \frac{m}{p_a}.
\end{aligned}$$

Interpretation is that if the decision maker spends all income m and spends nothing on good b (that is $x_b = 0$) then all income is spent on good a and the amount bought is $x_a = \frac{m}{p_a}$.

Answer to 1.3ii

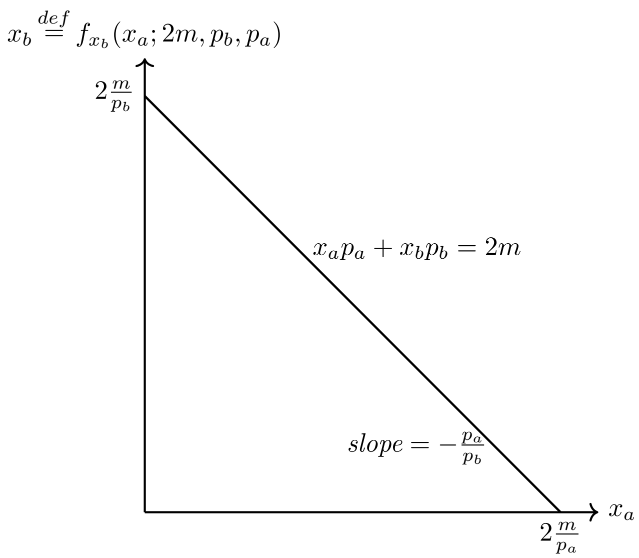
Now doubling of income is simply a change in one of the parameters, i.e. $m = 2m$. Then

$$f_{x_b}(x_a; 2m, p_b, p_a) = \frac{2m}{p_b} - \frac{p_a}{p_b}x_a = \underbrace{2\frac{m}{p_b}}_{\substack{\text{doubling of}\\ \text{a vertical}\\ \text{intercept}}} - \underbrace{\frac{p_a}{p_b}x_a}_{\substack{\text{no effect}\\ \text{on a slope}}}$$

Alternatively without treating x_b as a function the manipulations as in (NB) can be repeated:

$$\begin{aligned}
p_a x_a + p_b x_b &= 2m\\
p_b x_b &= 2m - p_a x_a\\
x_b &= \frac{2m}{p_b} - \frac{p_a}{p_b}x_a\\
x_b &= 2\frac{m}{p_b} - \frac{p_a}{p_b}x_a.
\end{aligned}$$

Either way it look like an outward shift of a budget line.



$x_b \stackrel{def}{=} f_{x_b}(x_a; 2m, p_b, p_a)$
$2\frac{m}{p_b}$
$x_a p_a + x_b p_b = 2m$
$slope = -\frac{p_a}{p_b}$
x_a
$2\frac{m}{p_a}$

Question 1

Consider the following expression:

$$x_a p_a + x_b p_b = m \tag{1}$$

where

- x_a and x_b denote the quantities of good a and b respectively;
- p_a and p_b denote the prices of goods a and b respectively;
- m denotes the income of a consumer.

1.4. Suppose $p_a = 2\$$ and $p_b = 6\$$. What is the opportunity cost of good a in terms of good b?

Answer to 1.4

The question may be easier to understand if it was rephrased as "how much of good b has to be abandoned to get a unit of good a".

Because good b is three time more expensive then the cost of good a, it is clear that the decision maker has to give up $\frac{1}{3}$ of good b to get 1 unit of good a. Which is the meaning of the slope $-\frac{p_a}{p_b} = -\frac{2}{6} = -\frac{1}{3}$.

The intuition is that the magnitude of the prices do not on their own affect the opportunity costs, what matters is a relationship between prices. Even for $p_a = 100$ and $p_b = 300$ the cost of a lost opportunity is the same.

Question 2

A household has to decide how much of this monthly income to spend on milk and how much on water.

- Suppose that this household has a fixed income of $300 per month to spend on these two goods.
- The price of milk is $3 per liter and the price of water is $2 per liter.
- Let x_1 denote liters of milk and x_2 liters of water.

2.1.
Write the mathematical expression of the choice set and that of the budget constraint.

Answer to 2.1

It is usually easier to rewrite verbal statement into a compact mathematical notation.
The variables:

$$x_1 = \text{Good 1} = \text{Milk}$$
$$x_2 = \text{Good 2} = \text{Water}.$$

The parameters:

$$m = 300 = \text{Income}$$
$$p_1 = 3 = \text{Price for milk}$$
$$p_2 = 2 = \text{Price for water}.$$

A parametric (with parameters $m = 300, p_1 = 3, p_2 = 2$) relationship between variables x_1 and x_2 is given by

$$\begin{aligned} p_1x_1 + p_2x_2 &= m \\ 3x_1 + 2x_2 &\leq 300 \leftarrow \text{choice set} \\ 3x_1 + 2x_2 &= 300 \leftarrow \text{budget constraint (budget line)}. \end{aligned}$$

Question 2

A household has to decide how much of this monthly income to spend on milk and how much on water.

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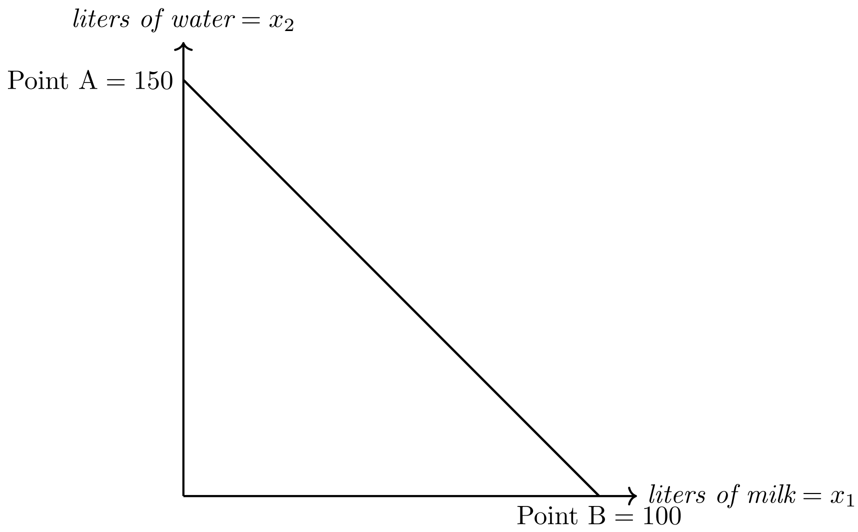
- Let x_1 denote liters of milk and x_2 liters of water.

2.2.

Represent the budget constraint faced by this family in a graph where you put "liters of milk" on the horizontal axis and "liters of water" on the vertical. In the graph, identify the values of the vertical and horizontal intercepts, and of the slope.

Answer to 2.2

The graph as has been already shown looks something like this:



The figure suggest that Point A is the maximum amount of purchased good 2 which happens when nothing is spent on good 1 and Point B is the maximum amount of purchased good 1 which happens when nothing is spent on good 2.

As before we can rearranging the budget line to express x_2 in terms of x_1 and other parameters:

$$\begin{aligned} 3x_1 + 2x_2 &= 300 \\ 2x_2 &= 300 - 3x_1 \\ x_2 &= 150 - \frac{3}{2}x_1. \end{aligned}$$

Then if $x_1 \stackrel{set}{=} 0$ then $x_2 = 150$, whereas if $x_2 \stackrel{set}{=} 0$ then $x_1 = 100$.

The identity also contains the slope of $-\frac{3}{2}$, which demonstrates how x_2 changes when x_1 changes.

An aside on variable/constant and opportunity costs

By the way, mathematically the slope is actually a partial derivative.

If x_2 again treated as a function

$$f_{x_2}(x_1; m, p_1, p_2) = \frac{m}{p_2} - \frac{p_1}{p_2} x_1$$

then

$$\frac{\partial}{\partial x_1} f_{x_2}(x_1; m, p_1, p_2) = -\frac{p_1}{p_2}.$$

The partial derivative states on how much x_2 changes when x_1 is increased. For this function it is negative, which is not surprising because money can buy you only one of two goods.

Also note that the partial derivative can also be treated as a function. Let us define

$$g_{x_2}(p_1, p_2) \stackrel{def}{=} \frac{\partial}{\partial x_1} f_{x_2}(x_1; m, p_1., p_2) = -\frac{p_1}{p_2}.$$

This function clearly does not depend on x_1 and m only on p_1 and p_2 which are two variables for this function.

Something that was considered parameters earlier now plays a role of variables. It reminds us that the classification is provisional and can't be well understood outside of a context.

Finally note that to get a particular solution a newly defined function needs to be evaluated for particular values of p_1 and p_2

$$g_{x_2}(3, 2) = -\frac{3}{2}.$$

Sometimes the following notation is used to stress that the function is evaluated at a particular values of variables.

$$g_{x_2}(p_1, p_2)\Bigg|_{p_1=3, p_2=2} = -\frac{3}{2}.$$

Question 2

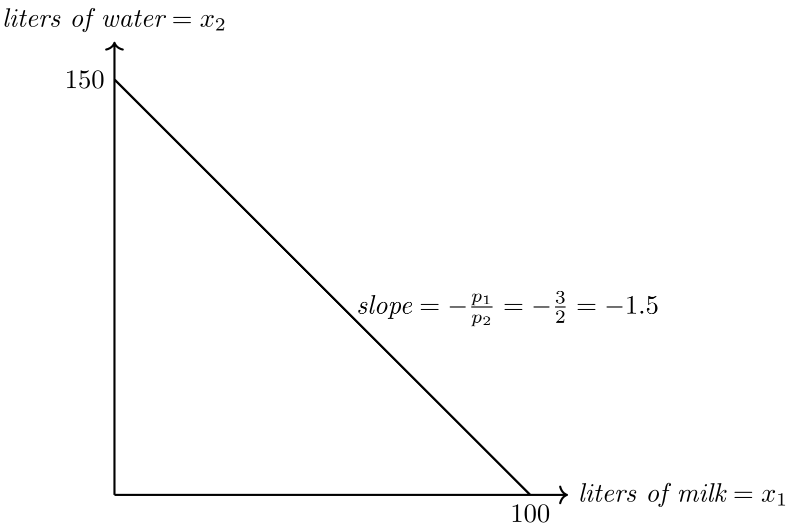
A household has to decide how much of this monthly income to spend on milk and how much on water.

- Suppose that this household has a fixed income of \$300 per month to spend on these two goods.
- The price of milk is \$3 per liter and the price of water is \$2 per liter.
- Let x_1 denote liters of milk and x_2 liters of water.

2.3.

Explain the economic meaning of the slope of the budget line that you have represented in the previous graph.

Answer to 2.3



The slope of the budget line indicates that for 1 additional liter of milk, the consumer would have to give up 1.5 liter of water.

Indeed 1 liter of milk costs \$3 which is equivalent to 1.5 liter of water

$$\text{Price of one liter of water} \times \text{Number of liters of water} = \text{Price of one liter of milk}$$
$$2 \times 1.5 = 3.$$

Question 2

A household has to decide how much of this monthly income to spend on milk and how much on water.

- Suppose that this household has a fixed income of $300 per month to spend on these two goods.
- The price of milk is $3 per liter and the price of water is $2 per liter.
- Let x_1 denote liters of milk and x_2 liters of water.

2.4.

Now suppose the government introduces a tax of $1 per liter of milk purchased.

- Represent graphically what happens to the choice set of the consumer after the introduction of the tax. (keep "liters of milk" on the horizontal axis and "liters of water" on the vertical).
- In the graph, show the new slope, horizontal and vertical intercept.

Answer to 2.4

This a good demonstration of how a mathematical model can be used for an economic policy analysis.

The variables of interest are the consumer choices x_1 and x_2 and by reasoning that the consumer operates within a choice set is enough to predict some outcomes of a policy change.

The policy affects the price of a good 1 which is one of the parameters of the models.

Let us define a new price p_1^t in term of an old price of good 1 and an imposed tax on good 1

$$p_1^t \overset{def}{=} p_1 + t.$$

Since a parametric relation between variables x_1 and x_2 which is $p_1 x_1 + p_2 x_2 = m$ holds for all p_1 and p_2 it also must hold for $p_1 = p_1^t$ and $p_2 = p_2$.

$$\underbrace{p_1^t}_{p_1+t} x_1 + p_2 x_2 = m$$

Because we have particular values for the parameters, i.e

$$\begin{aligned} m &= 300 \\ p_1 &= 3 \\ p_2 &= 2 \\ t &= 1, \end{aligned}$$

it is possible to find a particular solution.

Substituting the values gives the following:

$$p_1^t = 3 + 1 = 4$$

$$\underbrace{4}_{3+1} x_1 + 2x_2 = 300$$

To demonstrate it graphically in (x_1, x_2) space let us do a usual manipulates and express x_2 in term of everything else:

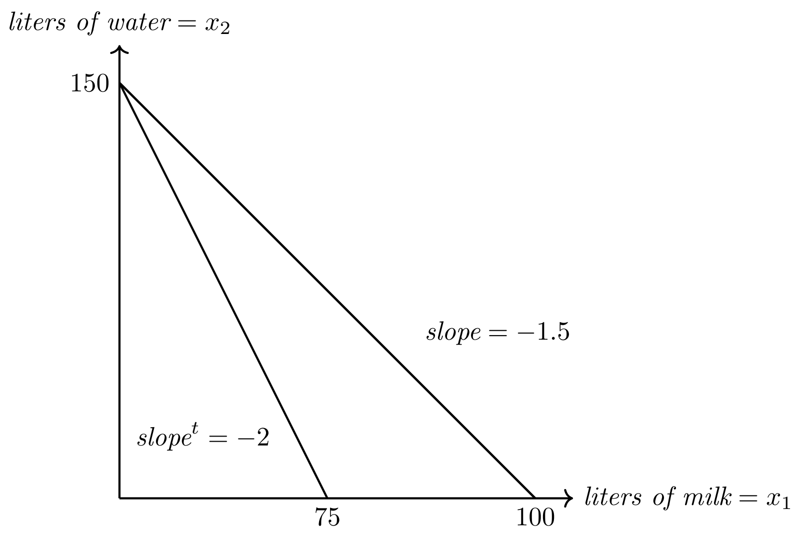
$$\begin{aligned} 4x_1 + 2x_2 &= 300 \\ 2x_2 &= 300 - 4x_1 \\ x_2 &= 150 - 2x_1 \end{aligned}$$

Then intercepts can be found as we already did earlier by setting $x_1 = 0$ and then $x_2 = 0$

$$x_1 \overset{set}{=} 0 \Rightarrow x_2 = 150$$

$$x_2 \overset{set}{=} 0 \Rightarrow x_1 = 75$$

Note that slope is steeper. Now you need even more water to exchange it for milk.



Question 2

A household has to decide how much of this monthly income to spend on milk and how much on water.

- Suppose that this household has a fixed income of \$300 per month to spend on these two goods.
- The price of milk is \$3 per liter and the price of water is \$2 per liter.
- Let x_1 denote liters of milk and x_2 liters of water.

2.5.

Now suppose that instead of taxing milk consumption, the government subsidizes it.

In particular, assume that the subsidy scheme works as follows:

- After buying 5 liters of milk at full price (i.e. \$3), the consumer gets a discount of 50% on each additional liter of milk that he/she buys.
- Represent graphically what happens to the choice set of the consumer after the introduction of this subsidy scheme (Keep "liters of milk" on the horizontal axis and "liters of water" on the vertical).
- In the graph, show the new slope, horizontal and vertical intercept.

Answer to 2.5

As usual, x_2 needs to be expressed in term of everything else. There is one observation, however.

Clearly the consumer can only buy $x_1 > 5$ if he/she already bought $x_1 = 5$ and spent $(5 \times 3 =)15$

We are ready to express x_2 as a function of x_1, m, p_1, p_2 depending on value of x_1.

$$\begin{aligned}
x_1 \leq 5 \Rightarrow 3x_1 + 2x_2 &= 300 \\
2x_2 &= 300 - 3x_1 \\
x_2 &= 150 - \frac{3}{2}x_1 \\
x_1 > 5 \Rightarrow 1.5x_1' + 2x_2' &= 285 \\
1.5x_1' + 2x_2' &= 285 \\
2x_2' &= 285 - 1.5x_1' \\
x_2' &= 142.5 - 0.75x_1'
\end{aligned}$$

The horizontal intercept then found by setting x_2' to 0

$$\begin{aligned}
x_2' \overset{set}{=} 0 \Rightarrow 0 &= 142.5 - 0.75x_1' \\
0.75x_1' &= 142.5 \\
x_1' &= 190
\end{aligned}$$

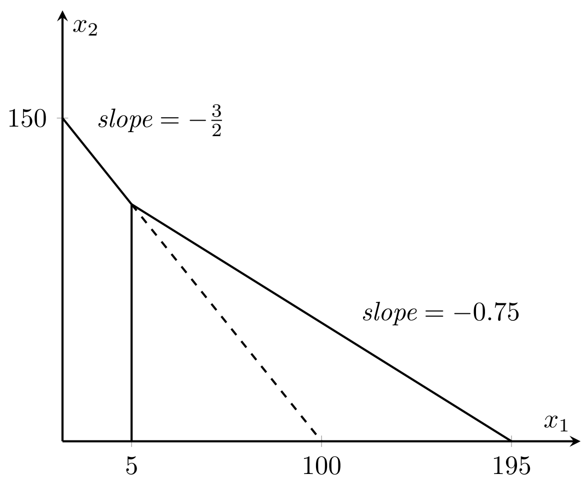
and by realizing that 5 units of x_1 was already bought

$$\begin{aligned}
x_1^{total} &= x_1' + 5 \\
x_1^{total} &= 190 + 5 \\
x_1^{total} &= 195
\end{aligned}$$

Vertical intercept stays unchanged as in the previous question.

Note that if x_2 is treated as a function then the figure visualizes the following function

$$f_{x_2}(x_1; m, p_1, p_2) = \begin{cases} 150 - \frac{3}{2}x_1 & \text{if } x_1 \leq 5 \\ 142.5 - 0.75x_1 & \text{if } x_1 > 5 \end{cases}$$


x_2
150
$slope = -\frac{3}{2}$
$slope = -0.75$
x_1
5
100
195

Tutorial 2 (27/03/2018)

Economic theory prescribes that consumers simply choose the bests thing they can afford.

Tutorial 1 clarified the meaning of **can afford**, this tutorial clarifies the meaning of **bests thing**

A recap on a choice set

An equation for a budget line

$$p_1x_1 + p_2x_2 = m$$

where

$$x_1 \text{ and } x_2 \text{ are variables}$$

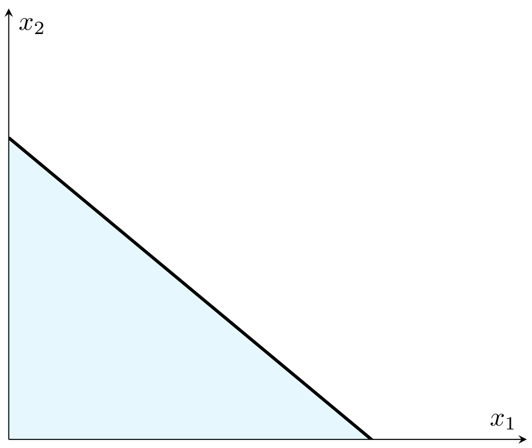
$$p_1 \text{ and } p_2 \text{ are parameters}$$

and

$$m \text{ is a constant}$$

usually m, p_1, p_2 are all called parameters

Graphical representation of the budget line



$$p_1x_1 + p_2x_2 = m \Leftrightarrow \text{a thick black line (budget line)}$$

$$p_1x_1 + p_2x_2 < m \Leftrightarrow \text{a blue area without the thick black line}$$

$$p_1x_1 + p_2x_2 \leq m \Leftrightarrow \text{both the thick black line and the blue area (choice set)}$$

Also recall

$$p_1 \uparrow \Rightarrow \text{choice set shrinks away from } x_1$$

$$p_2 \uparrow \Rightarrow \text{choice set shrinks away from } x_2$$

$$m \uparrow \Rightarrow \text{choice set expands preserving the shape}$$

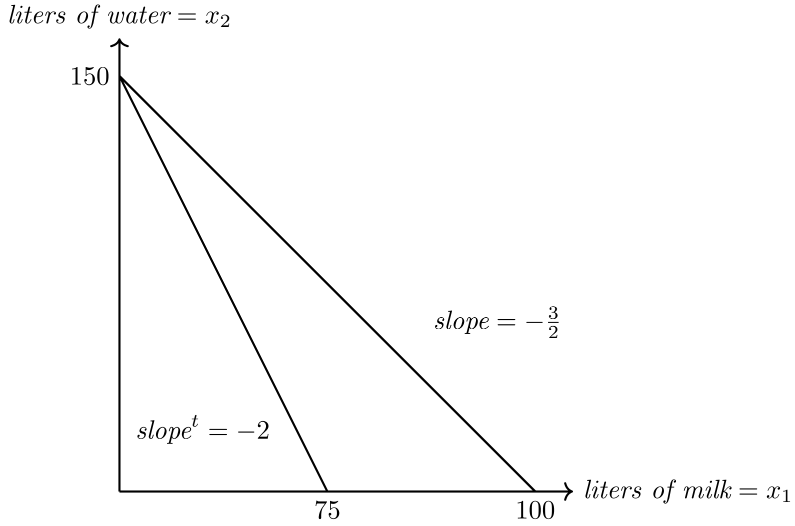
A recap on opportunity costs (OC) and meaning of the slope

Slope and OC

Slope indicates an OC of consuming milk in terms of not consumed water.

When the price on milk was 3 buying one litre of milk had an OC of $\frac{3}{2}$ litres of water.

When, due to tax, the price on milk increased till 4 the opportunity costs became 2 litres of water.



Intuition

Imagine that a young lady dates a guy A that does something she does not like, then she might say with disappointment to him that if she did not date him she would have dated another guy B.

In this context an opportunity cost of dating the guy A is (not) dating the guy B because just like money time spent on something never comes back.

OC without slope

	Before tax	After tax
Parameter	$p_{milk} = 3$ $p_{water} = 2$ $m = 300$	$p_{milk} = 4$ $p_{water} = 2$ $m = 300$
Thinking	Buying 1 litre of milk means saying no to $\frac{3}{2}$ liters of water	Buying 1 litre of milk means saying no to 2 liters of water
Reason	Because 1 litre of milk is 3$ of worth which in term of water is $\frac{3}{2}$ liters of water	Because 1 litre of milk is 3$ of worth which in term of water is 2 liters of water
Scale	To get the number we want to scale the worth of water to fit the worth of milk $p_{milk} = scale \times\ p_{water} \Rightarrow 3 = scale \times\ 2 \Rightarrow scale = \frac{3}{2}$	To get the number you want to scale the worth of water to fit the worth of milk $p_{milk} = scale \times\ p_{water} \Rightarrow 4 = scale \times\ 2 \Rightarrow scale = 2$
Slope	$3x_1 + 2x_2 = 300 \Rightarrow 2x_2 = 300 - 3x_1 \Rightarrow x_2 = 150 - \frac{3}{2}x_1$	$4x_1 + 2x_2 = 300 \Rightarrow 2x_2 = 300 - 4x_1 \Rightarrow x_2 = 150 - 2x_1$

A recap on a kinky budget set

Question 2

A household has to decide how much of this monthly income to spend on milk and how much on water.

- Suppose that this household has a fixed income of \$300 per month to spend on these two goods.
- The price of milk is \$3 per liter and the price of water is \$2 per liter.
- Let x_1 denote liters of milk and x_2 liters of water.

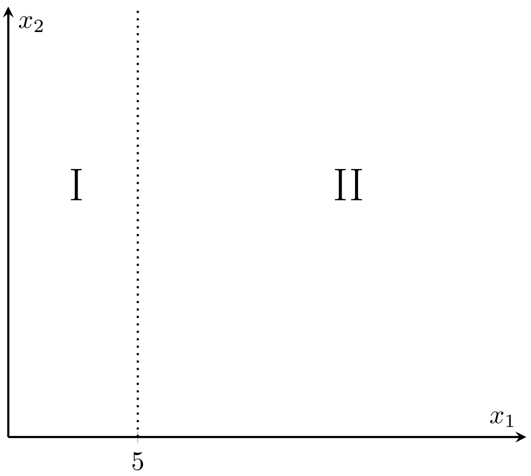
2.5.

Now suppose that instead of taxing milk consumption, the government subsidizes it.

In particular, assume that the subsidy scheme works as follows:

- After buying 5 liters of milk at full price (i.e. \$3), the consumer gets a discount of 50% on each additional liter of milk that he/she buys.
- Represent graphically what happens to the choice set of the consumer after the introduction of this subsidy scheme (Keep "liters of milk" on the horizontal axis and "liters of water" on the vertical).
- In the graph, show the new slope, horizontal and vertical intercept.

It is convenient to start with a figure and to divide it into to area I and II to the left and right of the value $x_1 = 5$



We have two sets of parameters depending on the area

Condition

to the left of 5 → $p_1^{\mathrm{I}} x_1^{\mathrm{I}} + p_2^{\mathrm{I}} x_2^{\mathrm{I}} = m^{\mathrm{I}}$

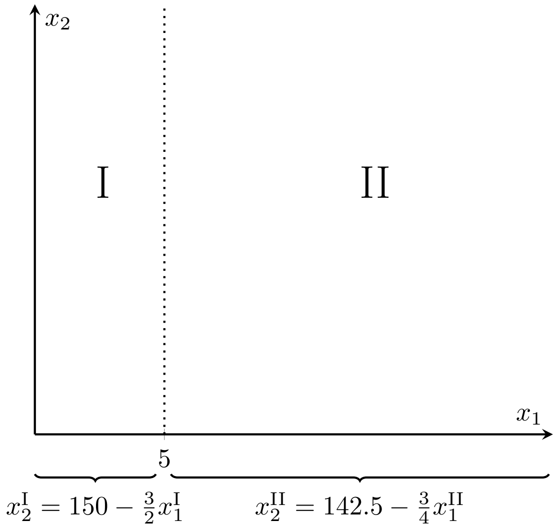
to the right of 5 → $p_1^{\mathrm{II}} x_1^{\mathrm{II}} + p_2^{\mathrm{II}} x_2^{\mathrm{II}} = m^{\mathrm{II}}$

From the question we know that the price of good 1 halves if more than 5 is bought

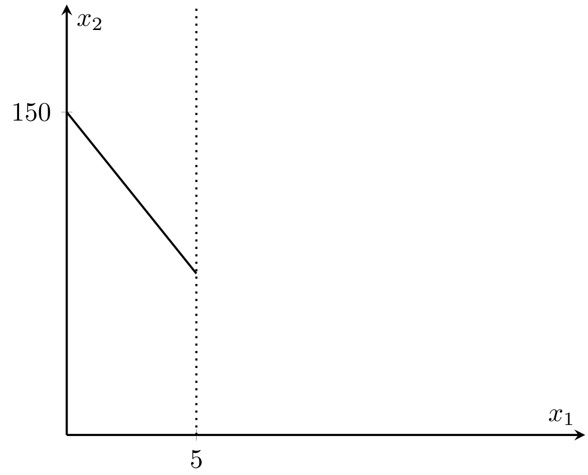
$$
\begin{aligned}
p_1^{\mathrm{I}} &= 3 \\
p_2^{\mathrm{I}} &= 2 \\
m^{\mathrm{I}} &= 300
\end{aligned}
\qquad\qquad
\begin{aligned}
p_1^{\mathrm{II}} &= \frac{3}{2} \\
p_2^{\mathrm{II}} &= 2
\end{aligned}
$$

Then

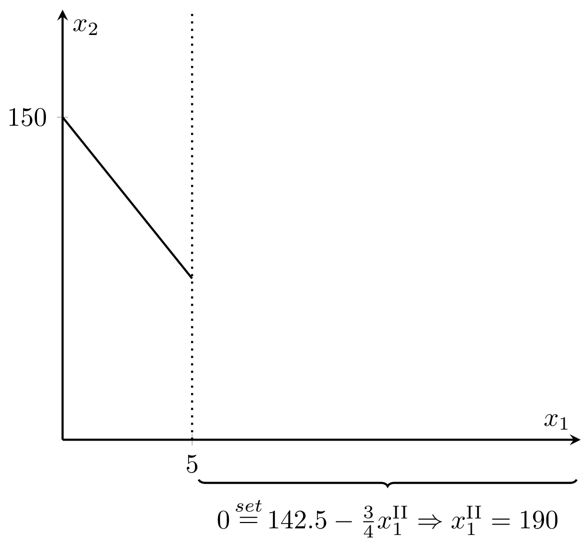
$$
\begin{aligned}
p_1^{\mathrm{I}} x_1^{\mathrm{I}} + p_2^{\mathrm{I}} x_2^{\mathrm{I}} &= m^{\mathrm{I}} \\
3x_1^{\mathrm{I}} + 2x_2^{\mathrm{I}} &= 300 \\
2x_2^{\mathrm{I}} &= 300 - 3x_1^{\mathrm{I}} \\
x_2^{\mathrm{I}} &= 150 - \frac{3}{2}x_1^{\mathrm{I}}
\end{aligned}
\qquad\qquad
\begin{aligned}
p_1^{\mathrm{II}} x_1^{\mathrm{II}} + p_2^{\mathrm{II}} x_2^{\mathrm{II}} &= m^{\mathrm{II}} \\
\frac{3}{2}x_1^{\mathrm{II}} + 2x_2^{\mathrm{II}} &= 300 - (3 \times 5) \\
2x_2^{\mathrm{II}} &= 285 - \frac{3}{2}x_1^{\mathrm{II}} \\
x_2^{\mathrm{II}} &= 142.5 - \frac{3}{4}x_1^{\mathrm{II}}
\end{aligned}
$$



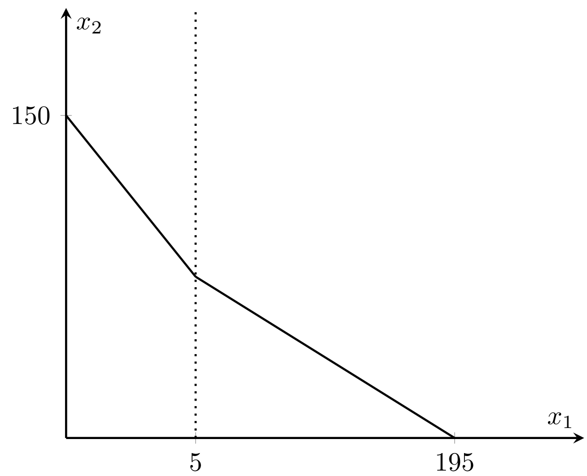
Where those equations belong on the figure



We can say straight away from the equation that to the left of point 5 the vertical intercept is 150



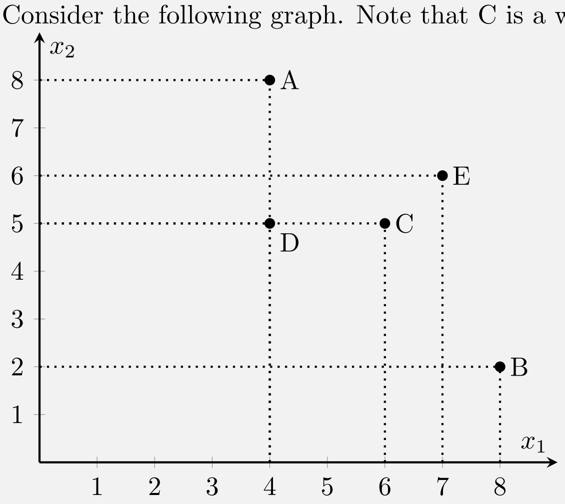
To find the horizontal intercept we set the equation for region II to 0, which gives us 190 and realize that we already had 5 which gives 195



Putting everything together gives us the following

Question 1

Consider the following graph. Note that C is a weighted average of A and B.



1.a.

Can you rank bundles A, B and C using only the monotonicity assumption?

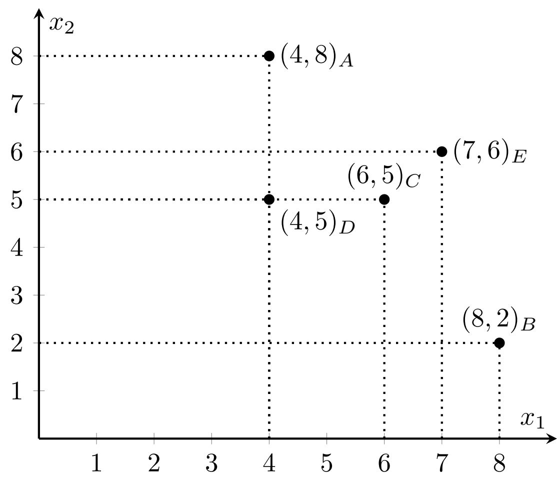
Answer to 1.a

In the question for notational convenience bundles defined as points in two dimensional space, e.g.

$$A \stackrel{def}{=} (x_1, x_2) = (4, 8)$$

But let's again redefine points on the figure to see the content of the bundle explicitly, e.g.

$$A \ = (4, 8) \stackrel{def}{=} (4, 8)_A$$



Now let us recall some operational notation that indicate a preference relations.

Imagine two arbitrary bundles with two goods

$$X = (x_1, x_2)_X \text{ and } Y = (y_1, y_2)_Y$$

then let us define

$$(x_1, x_2)_X \succ (y_1, y_2)_Y$$

which says that for the consumer X is *strictly preferred* to Y if we certain that X gives higher satisfaction

$$(x_1, x_2)_X \sim (y_1, y_2)_Y$$

which says the consumer is *indifferent* between X and Y, both give equal satisfaction (graphical interpretation of all the bundle the consumer is indifferent about gives a notion of *indifference curve*)

$$(x_1, x_2)_X \succsim (y_1, y_2)_Y$$

which says that for the consumer X is *weakly preferred* to Y, weakly means we can't tell for certain if the consumer receives equal, $X \sim Y$, or higher satisfaction, $X \succ Y$

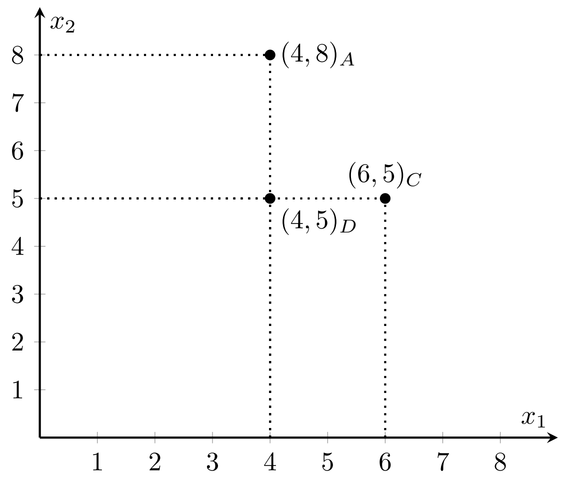
Note a dependency of those definitions, two examples

$$\left.\begin{array}{l}(x_1, x_2)_X \succsim (y_1, y_2)_Y \\ (y_1, y_2)_Y \succsim (x_1, x_2)_X\end{array}\right\} \Leftrightarrow (x_1, x_2)_X \sim (y_1, y_2)_Y$$

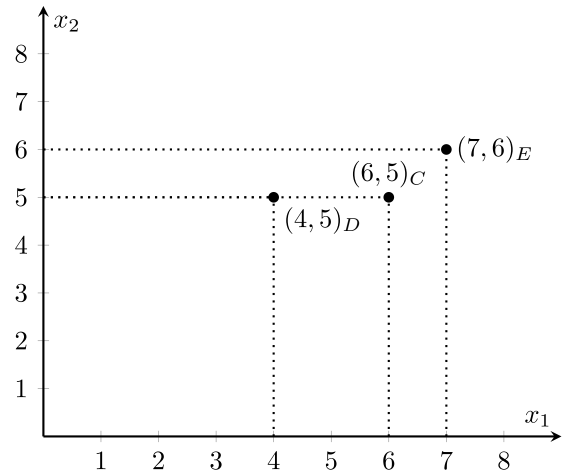
which says that if X and Y give equal satisfaction it is identical to say that X is weakly preferred to Y and at the same time Y is weakly preferred to X

$$\left.\begin{array}{l}(x_1, x_2)_X \succsim (y_1, y_2)_Y \\ (x_1, x_2)_X \not\sim (y_1, y_2)_Y\end{array}\right\} \Leftrightarrow (x_1, x_2)_X \succ (y_1, y_2)_Y$$

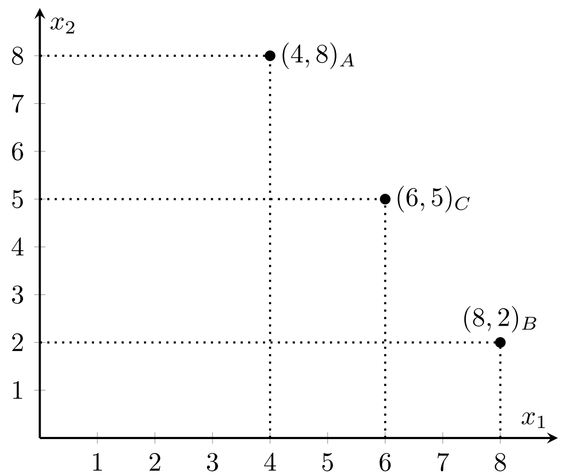
which says that if X is weakly preferred to Y and X is not indifferent to Y then X gives higher satisfaction



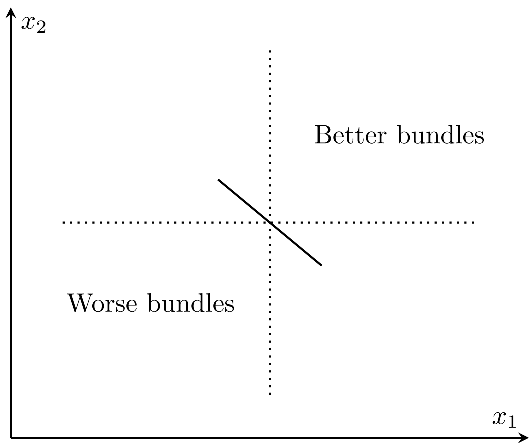
For microeconomics theory we reason that we weakly prefer the bundle that has more of one good and the same of the other, e.g. $(6,5)_C \succsim (4,5)_D$ and $(4,8)_A \succsim (4,5)_D$



and we also strictly prefer bundles that have more of both, e.g. $(7,6)_E \succ (6,5)_C$ and $(7,6)_E \succ (4,5)_D$ this property of preference relations is called *monotonicity*.



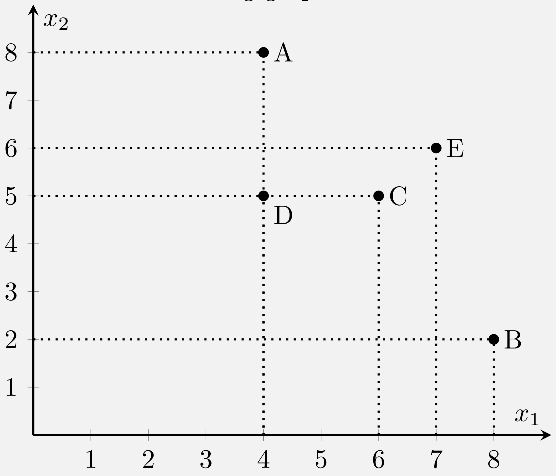
Note that monotonicity only won't allow us to to rank $(4,8)_A, (8,2)_B, (6,5)_C$ because an increase in one good always happens with a decrease in another



In general due to monotonicity assumption we like all bundles that are to the North-East which also explains why indifference curves have *negative* slopes.

Question 1

Consider the following graph. Note that C is a weighted average of A and B.



1.b.

Can you rank bundles A, B and C if you also assume that tastes satisfy convexity?

Answer to 1.b

We also reason that we prefer bundles with higher variety (average are preferred to extremes), i.e if

$$(x_1, x_2)_X \sim (y_1, y_2)_Y \tag{NB}$$

then

$$\left(\frac{1}{2}x_1 + \frac{1}{2}y_1, \frac{1}{2}x_2 + \frac{1}{2}y_2\right)_{\frac{1}{2}X+\frac{1}{2}Y} \succsim (x_1, x_2)_X$$

and

$$\left(\frac{1}{2}x_1 + \frac{1}{2}y_1, \frac{1}{2}x_2 + \frac{1}{2}y_2\right)_{\frac{1}{2}X+\frac{1}{2}Y} \succsim (y_1, y_2)_Y$$

Bundle $\frac{1}{2}X + \frac{1}{2}Y$ has an average amount of good 1 and an average amount of good 2. Graphically it gives a point that lies halfway along a straight line connecting $(x_1, x_2)_X$ and $(y_1, y_2)_Y$

In general, however, we like not only averaged mixtures but any mixture, i.e fix

$$t \in (0, 1) \Leftrightarrow 0 < t < 1$$

(in above we had $t = \frac{1}{2}$)

then if

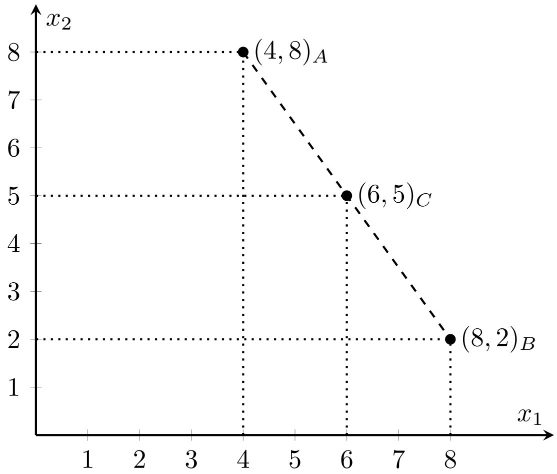
$$(x_1, x_2)_X \sim (y_1, y_2)_Y$$

then

$$(tx_1 + (1-t)y_1, tx_2 + (1-t)y_2)_{tX+(1-t)Y} \succsim (x_1, x_2)_X$$

Graphically this condition creates a line between $(x_1, x_2)_X$ and $(y_1, y_2)_Y$ and we prefer everything on that line to $(x_1, x_2)_X$ and $(y_1, y_2)_X$.

This property of a preference relation is called *convexity*.



Note that taking a half of $(4,8)_A$ and half of $(8,2)_B$ gives us point C

$$\left(\frac{1}{2}4+\frac{1}{2}8, \frac{1}{2}8+\frac{1}{2}2\right)_{\frac{1}{2}A+\frac{1}{2}B} = (6,5)_C$$

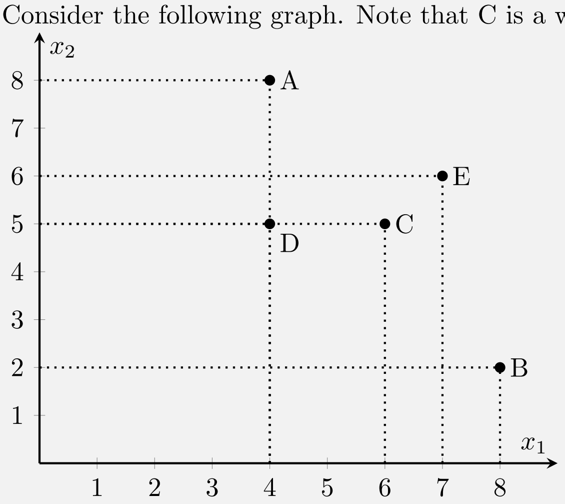
we can't, however, say anything about preference relation because we do not know if

$$(4,8)_A \sim (8,2)_B$$

which is a condition (NB)

Question 1

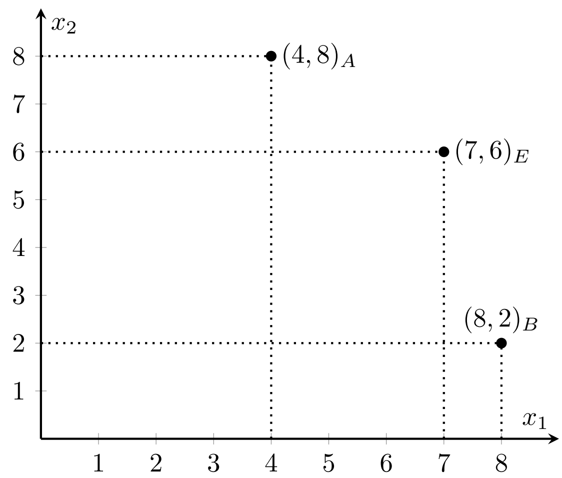
Consider the following graph. Note that C is a weighted average of A and B.



1.c.

Combining the convexity and monotonicity assumptions, can you now conclude something about the relationship between the pairs E and A and E and B?

Answer to 1.c



Points

$$(7, 6)_E$$

$$(4, 8)_A$$

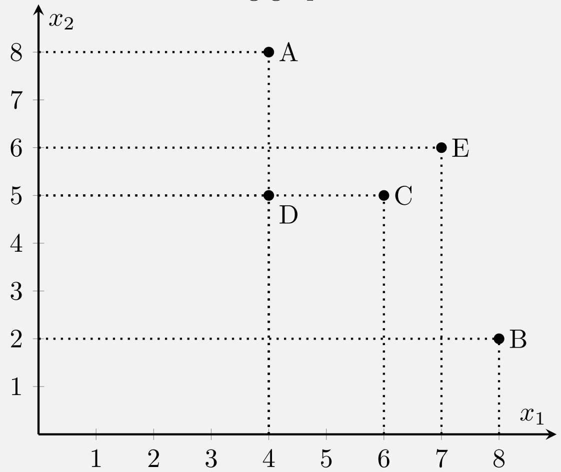
$$(8, 2)_B$$

are not in North-East region to each other, thus, monotonicity can't be applied.

We also do not know if the consumer is indifferent between any of the bundles, thus, convexity can't be applied either.

Question 1

Consider the following graph. Note that C is a weighted average of A and B.



1.d.

If you know that I am indifferent between A and B (and also that my tastes satisfy transitivity, monotonicity and convexity), can you conclude something about the relationship between the pairs E and A, and E and B?

Answer to 1.d

Recall that transitivity assumption is

$$\left.\begin{array}{l} X \succsim Y \\ Y \succsim Z \end{array}\right\} \overset{\text{transitivity}}{\Rightarrow} X \succsim Z$$

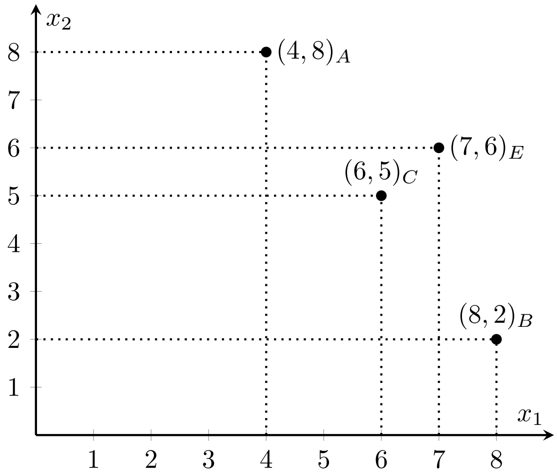
where as before we have bundles with two goods

$$X = (x_1, x_2)_X$$

$$Y = (y_1, y_2)_Y$$

$$Z = (z_1, z_2)_Z$$

In general transitivity preclude an intersection of indifference curves



$$(4,8)_A \overset{given}{\sim} (8,2)_B \overset{\text{convexity}}{\Rightarrow} \begin{array}{l}(6,5)_C \succsim (4,8)_A \\ (6,5)_C \succsim (8,2)_B\end{array} \overset{\text{monotonicity}}{\Rightarrow} (7,6)_E \succ (6,5)_C$$

$$\left.\begin{array}{l}(7,6)_E \succ (6,5)_C \\ (6,5)_C \succsim (4,8)_A\end{array}\right\} \overset{\text{transitivity}}{\Rightarrow} (7,6)_E \succsim (4,8)_A$$

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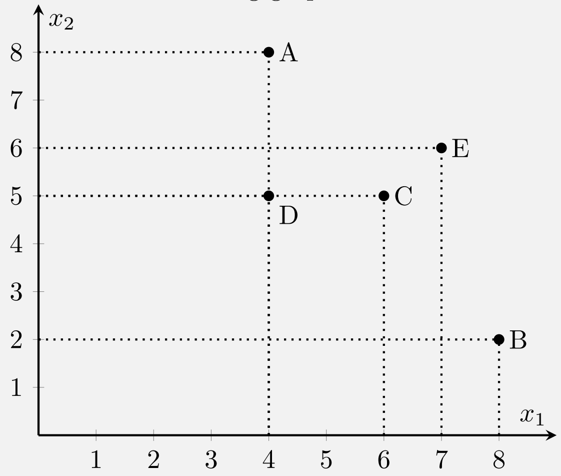
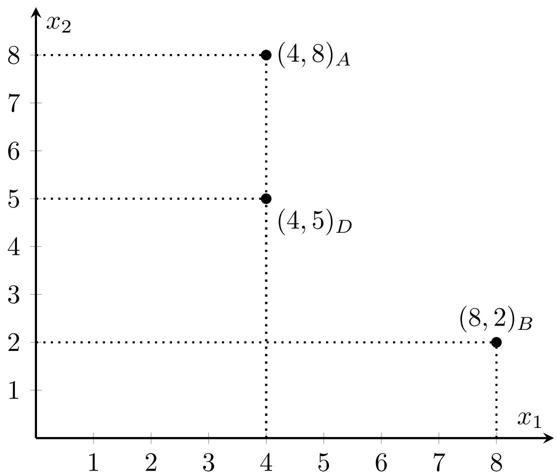


1.e.

Knowing that I am indifferent between A and B (and also that my tastes satisfy transitivity, monotonicity and convexity), can you now conclude something about how B and D are ranked by me? In order to reach this conclusion, do you necessarily have to invoke the convexity assumption?

Answer to 1.e



$$\left.\begin{array}{l} (8,2)_B \overset{\text{given}}{\sim} (4,8)_A \\ (4,8)_A \overset{\text{mono}}{\succsim} (4,5)_D \end{array}\right\} \overset{\text{transit}}{\Rightarrow} (8,2)_B \succsim (4,5)_D$$

Question 2

The assumption of monotonicity (i.e. "more is better") is what makes an item "a good" rather than "a bad". Indeed, if we assume that a consumer likes to have more of an item, that is because that item is good to him or her. But how do we deal with items that we do not like? In economics, items that a consumer does not like are called "bads". For a bad, the assumption of monotonicity is obviously violated. In fact, for a bad, the opposite holds: the less, the better. Now, here is the question.

Consider an individual with the following tastes:

- he likes to have more of good 1 (i.e. good 1 is a good for him), but less of good 2 (i.e. good 2 is a bad for him)
- for example, you can think of this individual being a gardener, good 1 being flowers, and good 2 being termites
- further, suppose that we know that the tastes of this individual also satisfy convexity

2.a.

Which of the standard assumptions about tastes is violated?

Answer to 2.a

Monotonicity is violated

Note that if we take a negative of good 2, i.e. the absence of termites, then monotonicity holds. That is a reasoning that is usually applied to satisfy the monotonicity assumption.

Question 2

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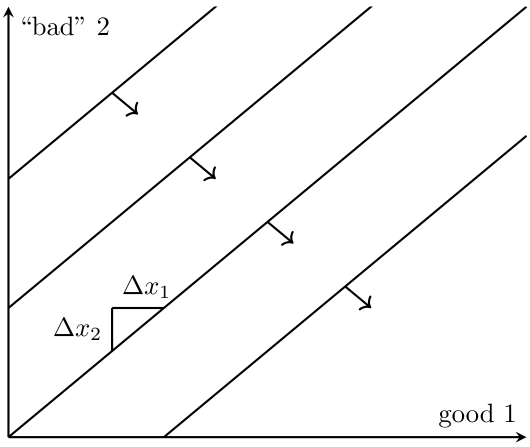
- further, suppose that we know that the tastes of this individual also satisfy convexity.

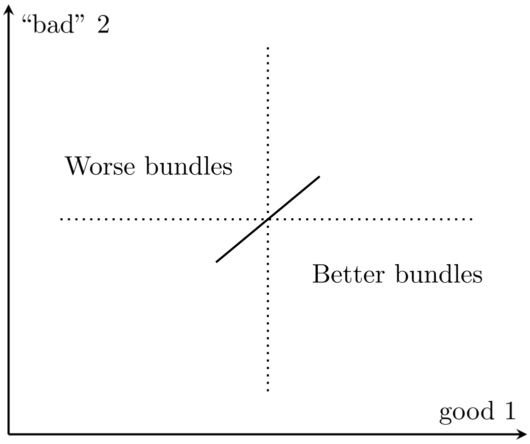
2.b.

- In a graph with good 1 on the horizontal axis and good 2 on the vertical, plot a map of indifference curves for this individual (assume that convexity is satisfied).
- With an arrow, show the direction where we find bundles that make this individual better off.

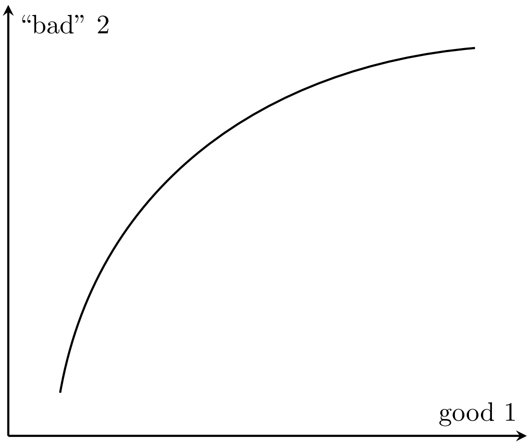
Answer to 2.b



A bad is a commodity that consumer does not like. Assuming a possibility of tradeoff between good 1 (which the guy wants) and good 2 (which the guys hates) means that there is exist some increase in the amount of good 1 that can compensate an amount of good 2. Keeping this reasoning in mind and ignoring for a moment the given condition of convexity allows to produce the following.



Another way to think about it is to realize that all bundle to South-East are better, whereas to North-West are worse. Thus, indifferent bundle got to be from South-West to North-East.



Convexity is the assumption that makes a set of all bundles that are weakly preferred look like a convex set. Or, put differently, convexity makes indifference curves bendy towards better bundles.

Question 2

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- further, suppose that we know that the tastes of this individual also satisfy convexity.

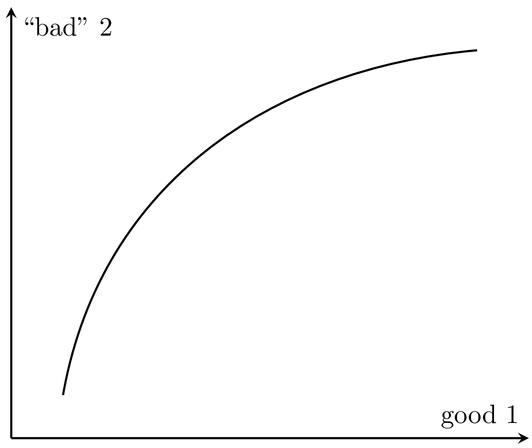
2.c.

Is the MRS negative or positive? How would you define the MRS in this case?

Answer to 2.c

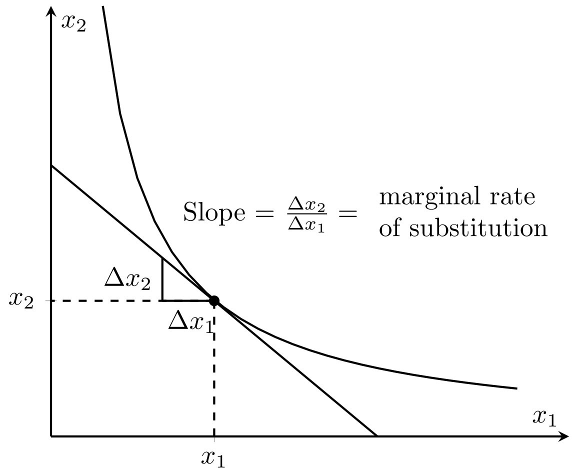
The slope of an indifference curve is also known as *marginal rate of substitution*. Name captures the fact that a consumer is willing to substitute one good for another.

With monotone goods MRS (the slope of the indifference curve) is negative. In this case it is positive and indicates the amount of good 2 (the "bad" good) the consumer is willing to accept for an additional unit of good 1 (the "good" good).



A recap on MRS

The slope of an indifference curve is also known as *marginal rate of substitution.* Name captures the fact that a consumer is willing to substitute one good for another.



MRS can be in interpreted as a trading at a particular exchange rate which has to be equal to MRS and at which the consumer wants to stay put.

It is useful to describe the the indifference curves by describing the behavior of MRS. For example, the "perfect substitutes" indifference curves are characterized by the fact that the MRS is constant at -1. The preferences for "perfect complements" are characterized by the fact that the MRS is either zero or infinity, and nothing in between.

Assumption of monotonicity implies that indifference curves must have a negative slope, so the MRS always involves reducing the consumption of one good in order to get more of another.

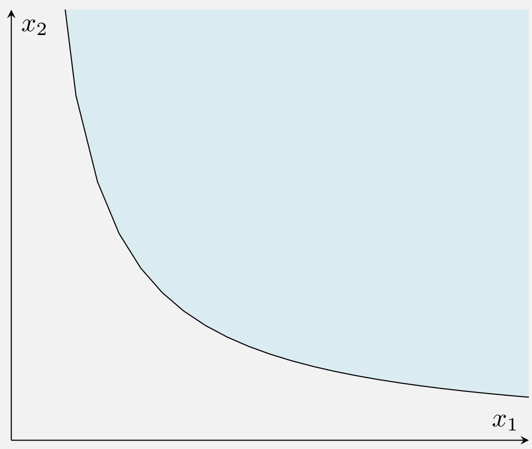
Assumption of convexity the MRS – the slope of the indifference curve – decreases (in absolute value) as we increase x_1. Thus the indifference curves exhibit a diminishing marginal rate of substitution.

This means that the amount of good 1 that the person is willing to give up for an additional amount of good 2 increases the amount of good 1 increases. Stated in this way, convexity of indifference curves seems very natural: it says that the more you have of one good, the more willing you are to give some of it up in exchange for the other good.

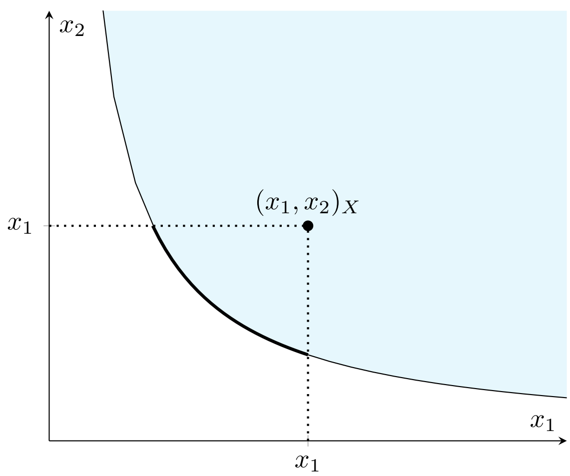
Remember that some times convexity is too much to ask. Imagine that goods are ice cream and olives you probably not gonna like the mix of those better. We can always reason, however, that you do not consume them at exact same moment, but spread consumption over, say, a week.

Question 3

- Assume that the 5 assumptions about tastes hold.
- Consider the graph below showing an indifference curve.
- Show that all bundles that lie to the north-east of the indifference curve (i.e. in the shaded area) are strictly preferred to **all** bundles that lie on the indifference curve.
- Note that showing this result proves that a consumer for which the assumption of monotonicity holds is better off as he moves toward indifference curves that lie in the north-east region of our graph (slide 25 of lecture 2).



Answer to 3



$$\left.\begin{array}{r} X \overset{\text{mono}}{\succ} \text{points on thick line} \\ \text{points on thick line} \overset{\text{given}}{\sim} \text{points on normal line} \end{array}\right\} \overset{\text{transit}}{\Rightarrow} X \succ \text{points on normal line}$$

To answer the question we only need assumption of monotonicity and transitivity.

It is a good place to recall all of the assumptions, though.

Five assumptions discussed in the textbooks are

- Completeness. Given any

$$(x_1, x_2)_X \text{ and } (y_1, y_2)_Y$$

we have

$$(x_1, x_2)_X \succsim (y_1, y_2)_Y \text{ or } (x_1, x_2)_X \precsim (y_1, y_2)_Y \text{ or both.}$$

Says that consumer is **able to compare**

- Transitivity. If

$$(x_1, x_2)_X \succsim (y_1, y_2)_Y \text{ and } (y_1, y_2)_Y \succsim (z_1, z_2)_Z$$

then

$$(x_1, x_2) \succsim (z_1, z_2)_Z.$$

Says that the consumer **won't "cycling"**, a convenient hypothesis about consumers' behavior (wait till question 4 to get it)

- Monotonicity. If

$$x_1 \geq y_1 \text{ and } x_2 \geq y_2$$

then

$$(x_1, x_2)_X \succsim (y_1, y_2)_Y$$

and if

$$x_1 > y_1 \text{ and } x_2 \geq y_2 \text{ or } x_1 \geq y_1 \text{ and } x_2 > y_2 \text{ or } x_1 > y_1 \text{ and } x_2 \geq y_2$$

then

$$(x_1, x_2)_X \succ (y_1, y_2.)_Y$$

Says that **the more the better**

- Convexity. For

$$t \in (0, 1)$$

and if

$$(x_1, x_2) \sim (y_1, y_2)$$

then

$$(tx_1 + (1-t)y_1, tx_2 + (1-t)y_2) \succsim (x_1, x_2).$$

Says that the consumer **loves variety**

- Continuity. Preference relation is continuous if for any sequence $\{x^n\} \subset X$ and $\{y^n\} \subset X$, then $n \in \mathbb{N}$ such that

$$\forall n, \quad x^n \succsim y^n$$

$$\lim_{n\to\infty} x^n = x, \quad \lim_{n\to\infty} y^n = y \quad (\text{where } x, y \in X)$$

then

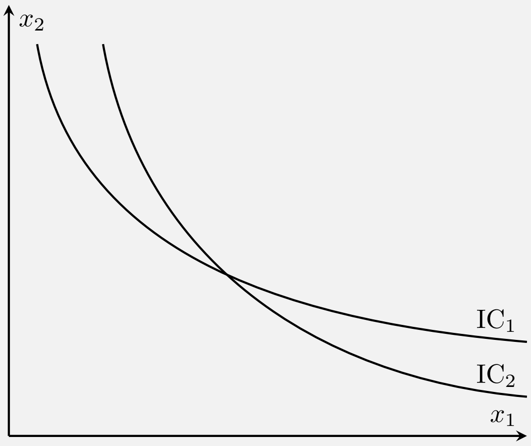
$$x \succsim y.$$

Says that **all goods can be divided into smaller pieces**, e.g. milk. Note that undividable goods can be soon as characteristics. (don' worry about this scary definition, you don't need it)

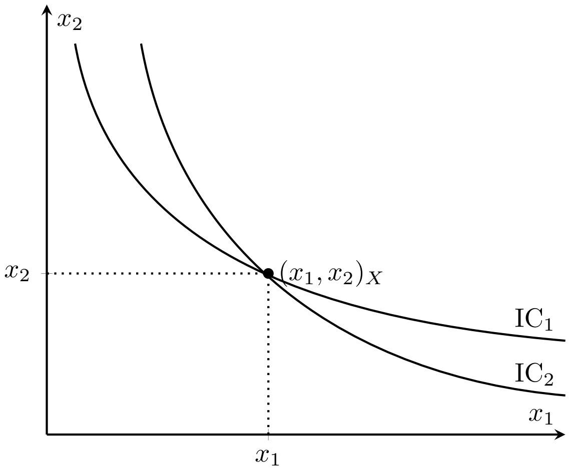
Let us return to answering the question

Question 4

- Suppose that the two indifference curves IC_1 and IC_2 are representing the tastes of the same individual over bundles of good 1 and good 2.
- Show that in this case the assumption of transitivity is necessarily violated.
- Note that proving this result is equivalent to prove the statement in slide 29 of lecture 2 that if the assumption of transitivity holds, then the ICs of the same individual cannot intersect.



Answer to 4



$$\left.\begin{array}{l} X \overset{\text{given}}{\sim} \text{points on IC}_1 \\ X \overset{\text{given}}{\sim} \text{points on IC}_2 \end{array}\right\} \overset{\text{transit}}{\Rightarrow} \text{points on IC}_1 \sim \text{points on IC}_2 \Rightarrow\Leftarrow$$

References

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