Proposed solutions for tutorial 5

*Intermediate Microeconomics (UTS 23567)**

Preliminary and incomplete

Available at https://backwardinduction.blog/tutoring/

Office hours on Mondays from 9 am till 10 am in building 8 on level 9

Please whatsapp me on 0457871540 so I could meet you at the door, I don't have an internal phone

Also please whatsapp if you have questions, I won't be able to answer through whatsapp but I will give answer during office hours or, since you are very unlikely to have ask a unique question, in the beginning of next tutorial

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Navigation

Page numbers are clickable

*Questions for the tutorial were provided by Massimo Scotti, slide and textbook are by Nechyba (2016). Solutions and commentary are by Sergey Alexeev (e-mail: sergei.v.alexeev@gmail.com). (Btw this is a much better book Varian (1987))

A price change in x_1 creates two effects

Say a guy has

$$u = x_1 x_2$$

then

if

$$2x_1 + x_2 = 10$$

the optimal bundle is

$$(2.5, 5) \equiv \underset{x_1, x_2}{\arg\max} \begin{cases} x_1 x_2 \\ 2x_1 + x_2 = 10 \end{cases}$$

let's call it $(2.5, 5)_A$

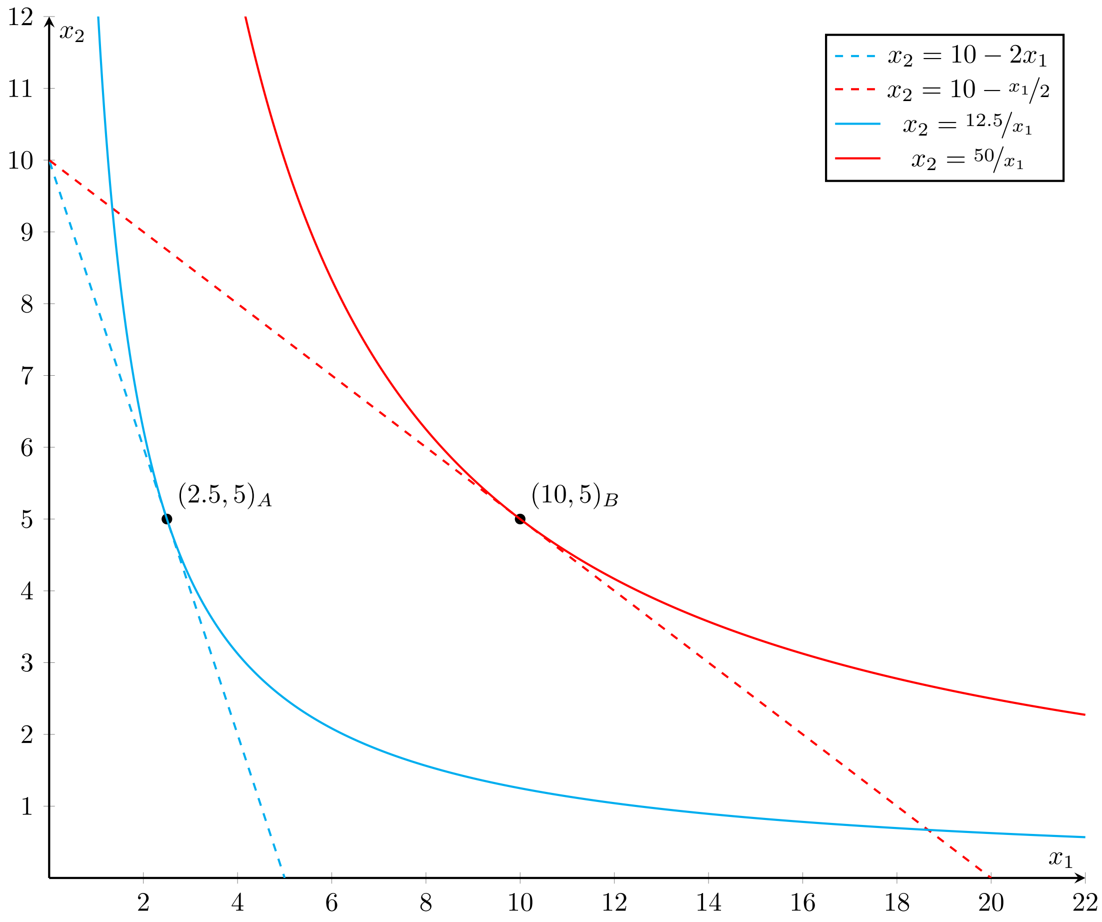
if

$$\frac{1}{2}x_1 + x_2 = 10$$

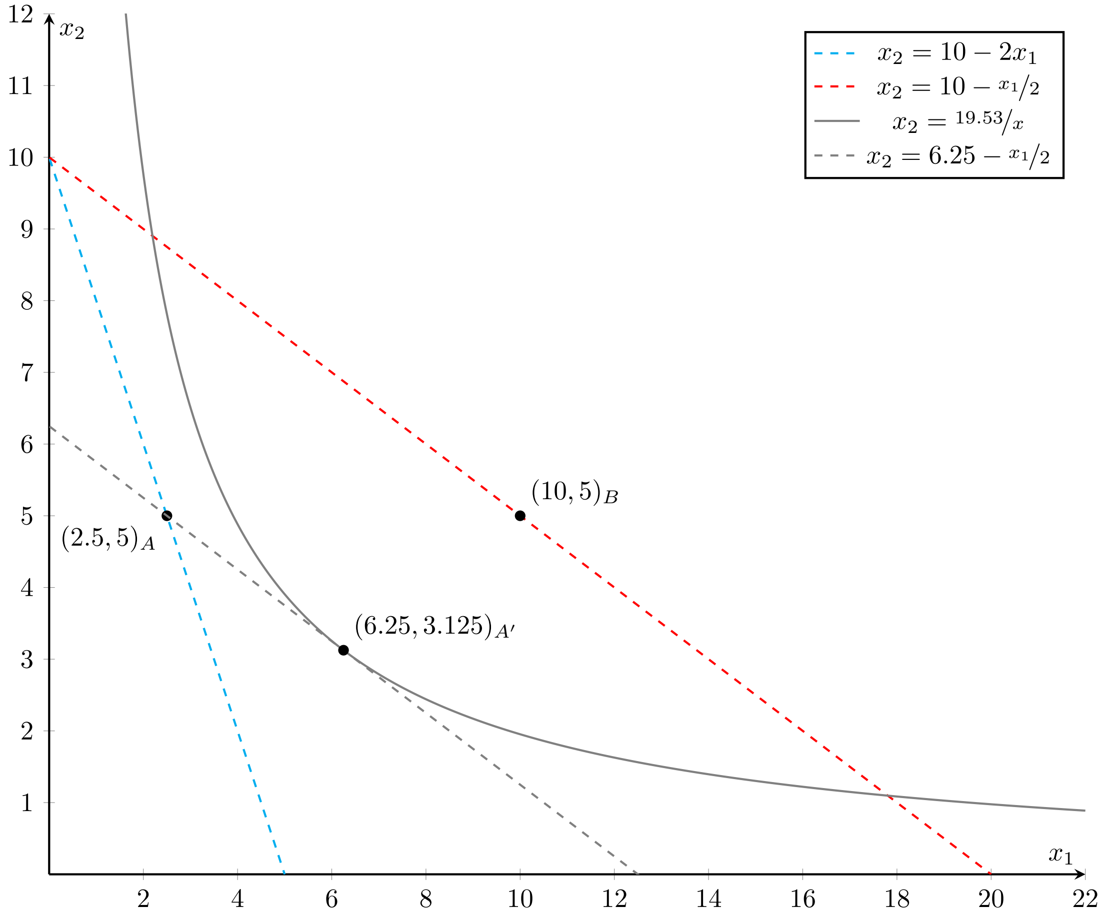
the optimal bundle is

$$(10, 5) \equiv \underset{x_1, x_2}{\arg\max} \begin{cases} x_1 x_2 \\ \frac{1}{2}x_1 + x_2 = 10 \end{cases}$$

let's call it $(10, 5)_B$



The change in demand due to the change in the rate of exchange between the two goods – the substitution effect



If prices are

$$\frac{1}{2}x_1 + x_2$$

and the guy is taken away

$$\Delta I = (2 - 0.5)2.5 = 3.75$$

then

$$(2.5, 5)$$

is still affordable, but not optimal because

$$(6.25, 3.125) \equiv \underset{x_1,x_2}{\arg\max} \begin{cases} x_1 x_2 \\ \frac{1}{2}x_1 + x_2 = 6.25 \end{cases}$$

is optimal. Let's call it A'

Then

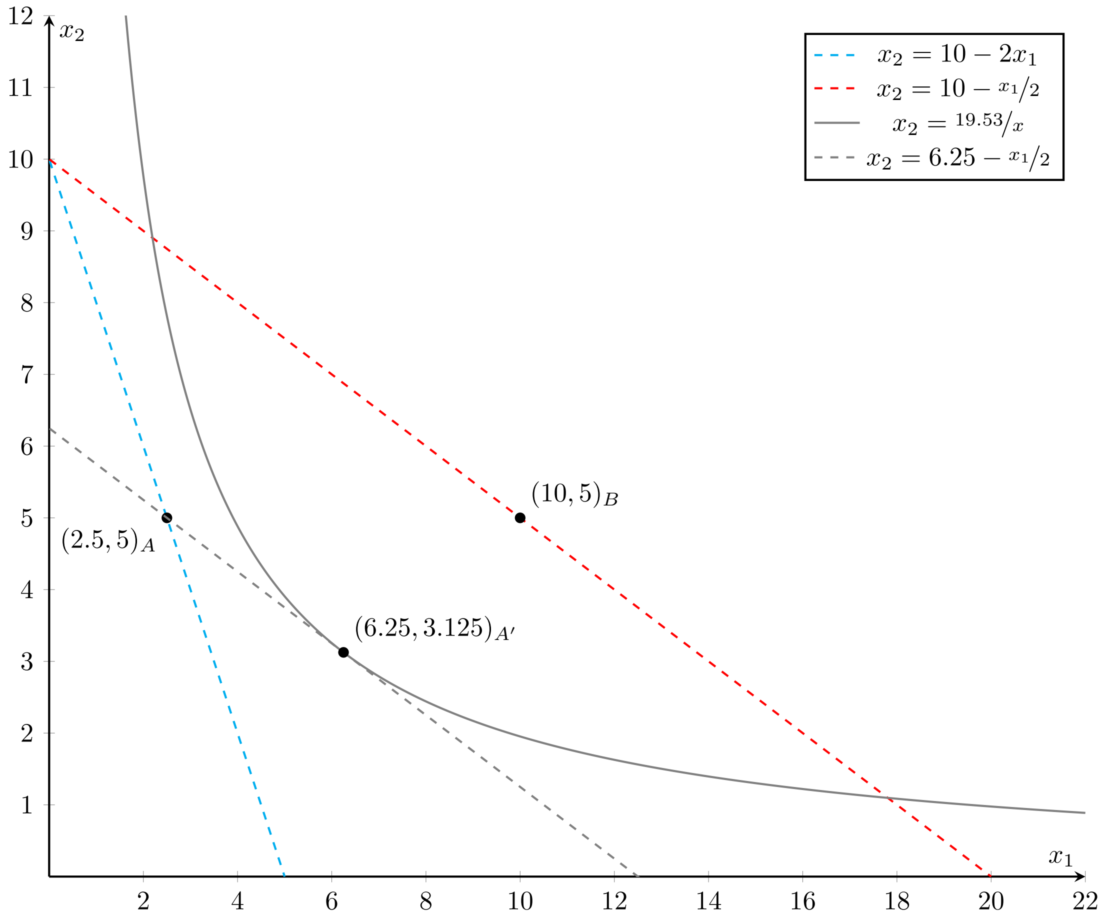
$$6.25 - 2.5 = 3.75$$

is a substitution effect

It indicates how the consumer "substitutes" one good for the other when a price changes but purchasing power remains constant.

$\boxed{\text{The substitution effect always moves opposite to the price movement. Substitution effect is negative}}$

The change in demand due to having more purchasing power – the income effect



If income is changed from

$$\frac{1}{2}x_1 + x_2 = 6.25$$

to

$$\frac{1}{2}x_1 + x_2 = 10$$

keeping the prices constant we have an income effect

$$10 - 6.25 = 3.75$$

$\boxed{\text{Income effect increases if good is a normal or decreases if good is inferior}}$

Total change it demand is a sum of income and substitution effects

$$3.75 + 3.75 = 7.5$$

We go through these troubles only to formulate Law of Demand

Microeconomics says that demand can go in any direction as circumstances change, which makes it a terrible science. Theory has to impose limitations (e.g. Law of Gravity), otherwise all of these math is for nothing. But now we know this:

If

$$x_1 \text{ is a normal good}$$

and

$$p_1 \uparrow$$

then

$$\underset{(-)}{\Delta x_1} = \underset{(-)}{\Delta x_1^s} + \underset{(-)}{\Delta x_1^n}$$

substitution effect Δx_1^s and income effect Δx_1^n reinforce each other

If

$$x_1 \text{ is an inferior good}$$

and

$$p_1 \uparrow$$

then

$$\underset{(?)}{\Delta x_1} = \underset{(-)}{\Delta x_1^s} + \underset{(+)}{\Delta x_1^n}$$

substitution effect Δx_1^s and income effect Δx_1^n opposite each other

If the income effect Δx_1^n is larger Δx_1^s, the total change in demand is positive.

Which is weird.

Such a good called Giffen.

A Giffens good could be understood as a very very inferior good.

Note that if we already know that the demand increases when income increases

$$\underset{(-)}{\Delta x_1^n}$$

i.e. the good is normal

then the substitution effect and the income effect reinforce each other, and an increase in price reduces demand.

This logic is known as Law of Demand

$$\boxed{\text{If the demand for a good increases when income increases, then the demand for that good must decrease when its price increases.}}$$

It follows directly from the Slutsky equation.

However, you are told that a good satisfies Law of Demand if

$$p_1 \uparrow \;\Rightarrow\; x_1 \downarrow$$

and

$$p_1 \downarrow \;\Rightarrow\; x_1 \uparrow$$

which is a good enough explanation

Tutorial 5 (17/04/2018)

Question 1

- Suppose you find that income and substitution effects for Good 1 go in opposite direction when the own price of Good 1, p_1 changes.
- Suppose you also find that the income effect is smaller than the substitution effect.

1.a.

Does Good 1 satisfy the law of demand?

Answer to 1.a

Say

$$p_1 \uparrow$$

We know

$$\underset{(?)}{\Delta x_1} = \underset{(-)}{\Delta x_1^s} + \underset{(?)}{\Delta x_1^n}$$

and we are also told that income effect and substitution effect have opposite signs

$$\underset{(?)}{\Delta x_1} = \underset{(-)}{\Delta x_1^s} + \underset{(+)}{\Delta x_1^n}$$

and that

$$\underset{(+)}{\Delta x_1^n} < \underset{(-)}{\Delta x_1^s}$$

clearly

$$\underset{(-)}{\Delta x_1} = \underset{(-)}{\Delta x_1^s} + \underset{(+)}{\Delta x_1^n}$$

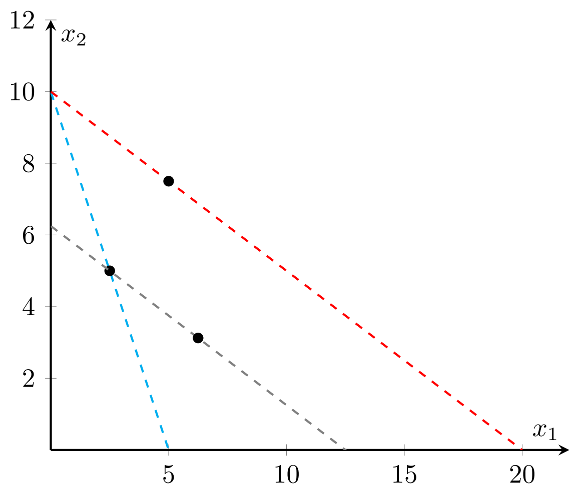
In short we have

$$p_1 \uparrow \;\Rightarrow\; x_1 \downarrow$$

by symmetry

$$p_1 \downarrow \;\Rightarrow\; x_1 \uparrow$$

which indicates that law of demand holds



Question 1

- Suppose you find that income and substitution effects for Good 1 go in opposite direction when the own price of Good 1, p_1 changes.
- Suppose you also find that the income effect is smaller than the substitution effect.

1.b

What is good 1 called?

Answer to 1.b

In situation like this

$$\underset{(-)}{\Delta x_1} = \underset{(-)}{\Delta x_1^s} + \underset{(+)}{\Delta x_1^n}$$

the good is inferior, but not too inferior to make it a Giffen good.

Question 1 Checklist

- Suppose you find that income and substitution effects for Good 1 go in opposite direction when the own price of Good 1, p_1 changes.
- Suppose you also find that the income effect is smaller than the substitution effect.

1.a.

Does Good 1 satisfy the law of demand?

1.b

What is good 1 called?

Question 2

- Suppose there are only two goods, Good 1 and Good 2.
- Further, suppose that Good 2 is an inferior good.
- Then, it must be that an increase in the price of Good 1, p_1 leads on an increase in the consumption of Good 2.
- True or False? Explain.

Answer to 2

Take

$$\underset{(?)}{\Delta x_2} = \underset{(?)}{\Delta x_2^s} + \underset{(?)}{\Delta x_2^n}$$

We know

$$p_1 \uparrow \; \rightarrow \; I \downarrow$$

because x_2 is inferior

$$I \downarrow \; \Rightarrow \; \underset{(+)}{\Delta x_2^n}$$

and consumer also starts substituting with x_2

$$p_1 \uparrow \; \Rightarrow \; \underset{(+)}{\Delta x_2^s}$$

Taken together

$$\underset{(+)}{\Delta x_2} = \underset{(+)}{\Delta x_2^s} + \underset{(+)}{\Delta x_2^n}$$

Question 2 Checklist

- Suppose there are only two goods, Good 1 and Good 2.
- Further, suppose that Good 2 is an inferior good.
- Then, it must be that an increase in the price of Good 1, p_1 leads on an increase in the consumption of Good 2.

True or False? Explain.

Question 3

- Suppose that a consumer has preferences described by the following utility function:

$$u(x_1, x_2) = x_1 x_2$$

- Assume that we put x_1 on the horizontal axis and x_2 on the vertical axis.
- Also, let p_1, p_2 and I respectively denote the price of x_1, the price of x_2 and the income of the consumer.

3.a

Find the demand functions for good 1 and good 2.

Answer to 3.a

$$\begin{cases} MRS(x_1, x_2) & = -{}^{p_1}/_{p_2} \\ p_1 x_1 + p_2 x_2 & = I \end{cases} \Rightarrow \begin{cases} -{}^{x_2}/_{x_1} & = -{}^{p_1}/_{p_2} \\ p_1 x_1 + p_2 x_2 & = I \end{cases}$$

$$\Rightarrow \begin{cases} x_1 p_1 & = x_2 p_2 \\ p_1 x_1 + p_2 x_2 & = I \end{cases}$$

$$\Rightarrow \begin{cases} x_1 p_1 & = x_2 p_2 \\ p_1 x_1 + p_1 x_1 & = I \end{cases}$$

$$\Rightarrow \begin{cases} x_1 p_1 & = x_2 p_2 \\ 2 p_1 x_1 & = I \end{cases}$$

$$\Rightarrow \begin{cases} x_1 p_1 & = x_2 p_2 \\ x_1 & = {}^{I}/_{2p_1} \end{cases}$$

$$\Rightarrow \begin{cases} x_1 & = {}^{I}/_{2p_1} \\ x_2 & = {}^{I}/_{2p_2} \end{cases}$$

Question 3

- Suppose that a consumer has preferences described by the following utility function:

$$u(x_1, x_2) = x_1 x_2$$

- Assume that we put x_1 on the horizontal axis and x_2 on the vertical axis.
- Also, let p_1, p_2 and I respectively denote the price of x_1, the price of x_2 and the income of the consumer.

3.b

Suppose

$$p_1 = 1$$

$$p_2 = 3$$

and

$$I = 90$$

Find the optimal bundle chosen by the consumer and provide a graphical representation.

Answer to 3.b

A particular solution with given parameters

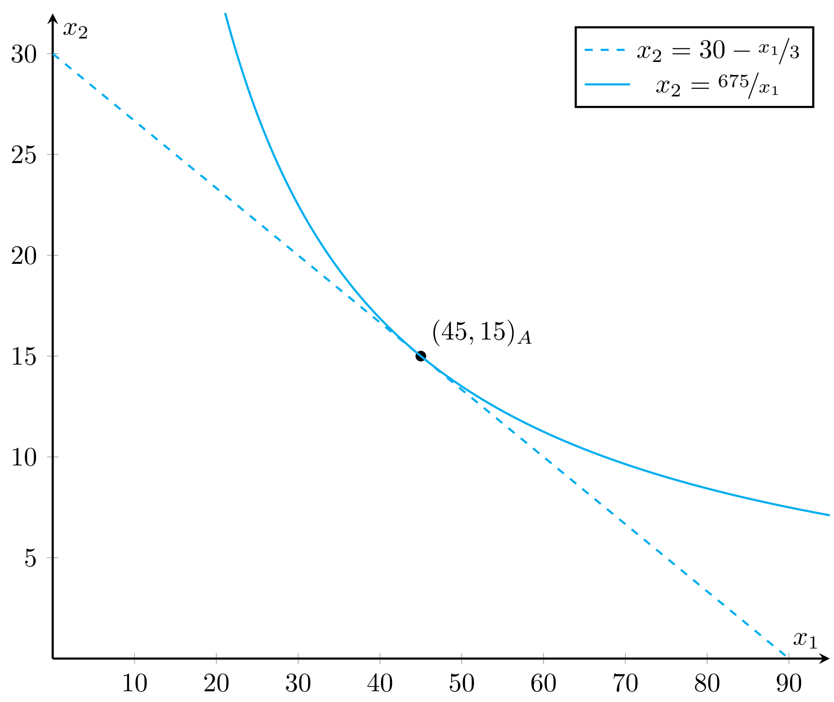
$$\begin{cases} x_1 &= I/2p_1 \\ x_2 &= I/2p_2 \end{cases} \Rightarrow \begin{cases} x_1 &= 45 \\ x_2 &= 15 \end{cases}$$

To plot the budget line

$$\begin{aligned} x_1 + 3x_2 &= 90 \\ 3x_2 &= 90 - x_1 \\ x_2 &= 30 - x_1/3 \end{aligned}$$

To plot the indifference curve

$$\begin{aligned} x_1 x_2\Big|_{(45,15)} &= 675 \\ x_2 &= 675/x_1 \end{aligned}$$



Question 3

- Suppose that a consumer has preferences described by the following utility function:

$$u(x_1, x_2) = x_1 x_2$$

- Assume that we put x_1 on the horizontal axis and x_2 on the vertical axis.
- Also, let p_1, p_2 and I respectively denote the price of x_1, the price of x_2 and the income of the consumer.

3.c

Suppose p_1 increases to 9. How does the optimal bundle change?

In the same graph you have used for part b, provide a graphical representation of how the optimal bundle changes.

Answer to 3.c

A particular solution with given parameters

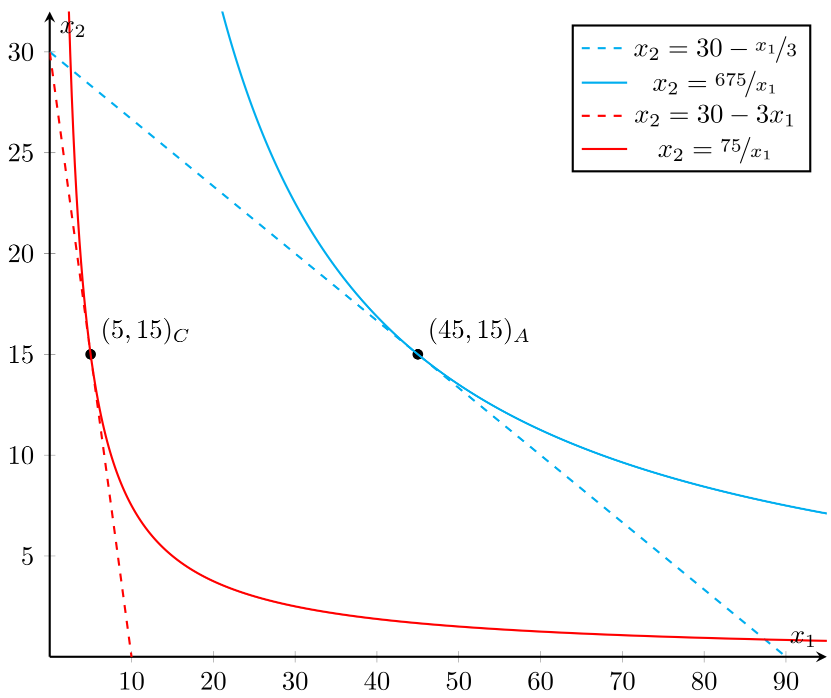
$$\begin{cases} x_1 &= I/2p_1 \\ x_2 &= I/2p_2 \end{cases} \Rightarrow \begin{cases} x_1 &= 5 \\ x_2 &= 15 \end{cases}$$

To plot the budget line

$$\begin{aligned} 9x_1 + 3x_2 &= 90 \\ 3x_2 &= 90 - 9x_1 \\ x_2 &= 30 - 3x_1 \end{aligned}$$

To plot the indifference curve

$$\begin{aligned} x_1 x_2 \Big|_{(5,15)} &= 75 \\ x_2 &= 75/x_1 \end{aligned}$$



Question 3

- Suppose that a consumer has preferences described by the following utility function:

$$u(x_1, x_2) = x_1 x_2$$

- Assume that we put x_1 on the horizontal axis and x_2 on the vertical axis.
- Also, let p_1, p_2 and I respectively denote the price of x_1, the price of x_2 and the income of the consumer.

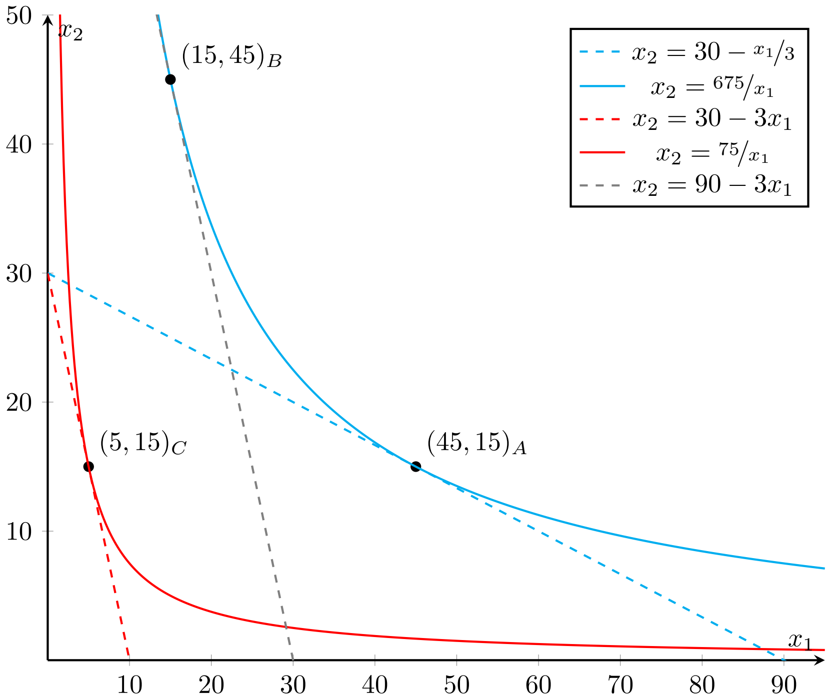
3.d

Find the size and direction of the income and substitution effects for good 1 and for good 2?

[Note: You are require to provide a numerical answer]

Answer to 3.d

$$\begin{cases} u(x_1,x_2)|_B &= u(x_1,x_2)|_A \\ MRS(x_1,x_2)|_B &= -{}^{p_1^{new}}/_{p_2^{new}} \end{cases} \Rightarrow \begin{cases} x_1x_2 &= 675 \\ -{}^{x_2}/_{x_1} &= -{}^{9}/_{3} \end{cases}$$
$$\Rightarrow \begin{cases} x_1x_2 &= 675 \\ 9x_1 &= 3x_2 \end{cases}$$
$$\Rightarrow \begin{cases} x_1x_2 &= 675 \\ 3x_1 &= x_2 \end{cases}$$
$$\Rightarrow \begin{cases} 3x_1x_1 &= 675 \\ x_2 &= 3x_1 \end{cases}$$
$$\Rightarrow \begin{cases} x_1^2 &= 225 \\ 3x_1 &= x_2 \end{cases}$$
$$\Rightarrow \begin{cases} x_1 &= 15 \\ x_2 &= 45 \end{cases}$$



$$p_1 \uparrow$$

$$(45, 15)_A \to (15, 45)_B \to (5, 15)_C$$

$$\underset{(-40)}{\Delta x_1} = \underset{(-30)}{\Delta x_1^s} + \underset{(-10)}{\Delta x_1^n}$$

$$p_2 \sim$$

$$(45, 15)_A \to (15, 45)_B \to (5, 15)_C$$

$$\underset{(0)}{\Delta x_2} = \underset{(+30)}{\Delta x_2^s} + \underset{(-30)}{\Delta x_2^n}$$

Question 3

- Suppose that a consumer has preferences described by the following utility function:

$$u(x_1, x_2) = x_1 x_2$$

- Assume that we put x_1 on the horizontal axis and x_2 on the vertical axis.
- Also, let p_1, p_2 and I respectively denote the price of x_1, the price of x_2 and the income of the consumer.

3.e

Is good 1 normal, inferior or quasi-linear?

Answer to 3.e

$$p_1 \uparrow$$

$$(45, 15)_A \rightarrow (15, 45)_B \rightarrow (5, 15)_C$$

$$\underset{(-40)}{\Delta x_1} = \underset{(-30)}{\Delta x_1^s} + \underset{(-10)}{\Delta x_1^n}$$

$$x_1 = \frac{I}{2p_1}$$

Question 3

- Suppose that a consumer has preferences described by the following utility function:

$$u(x_1, x_2) = x_1 x_2$$

- Assume that we put x_1 on the horizontal axis and x_2 on the vertical axis.
- Also, let p_1 , p_2 and I respectively denote the price of x_1, the price of x_2 and the income of the consumer.

3.f

Is good 2 normal, inferior or quasi-linear?

Answer to 3.f

$$p_2 \sim$$

$$(45, 15)_A \rightarrow (15, 45)_B \rightarrow (5, 15)_C$$

$$\underset{(0)}{\Delta x_2} = \underset{(+30)}{\Delta x_2^s} + \underset{(-30)}{\Delta x_2^n}$$

$$x_2 = \frac{I}{2p_2}$$

Question 3

- Suppose that a consumer has preferences described by the following utility function:

$$u(x_1, x_2) = x_1 x_2$$

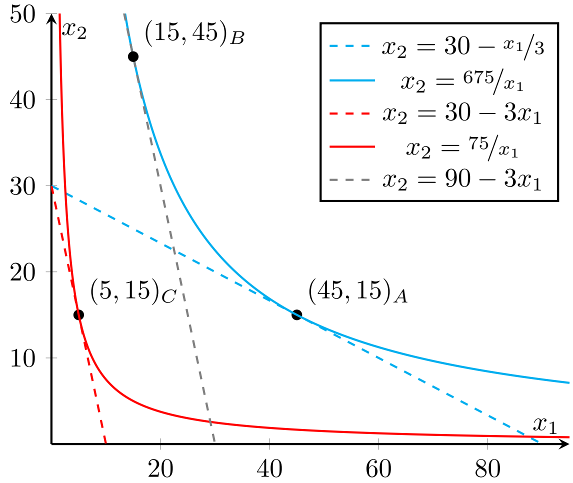
- Assume that we put x_1 on the horizontal axis and x_2 on the vertical axis.
- Also, let p_1, p_2 and I respectively denote the price of x_1, the price of x_2 and the income of the consumer.

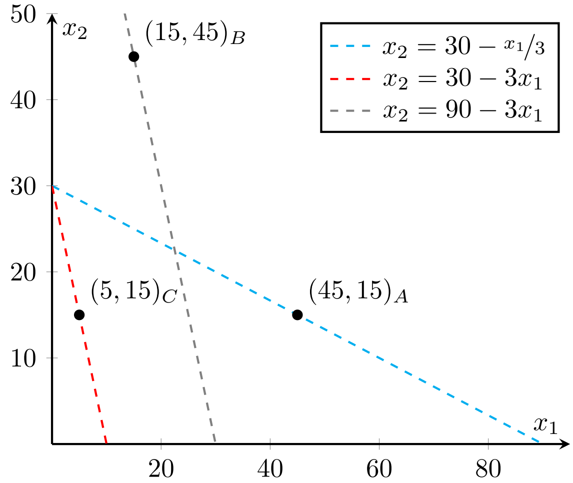
3.g

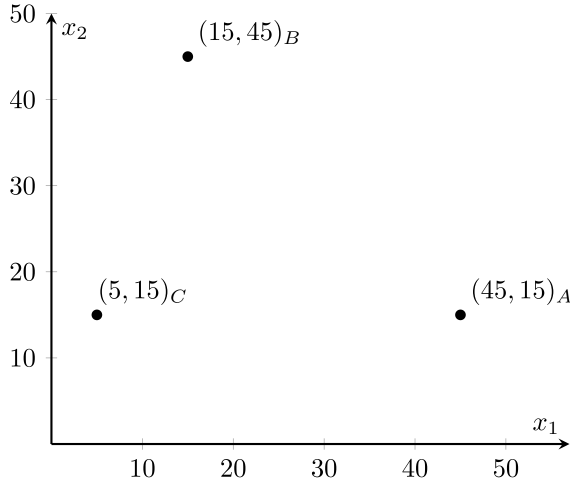
In the same graph you have used for parts b and c:

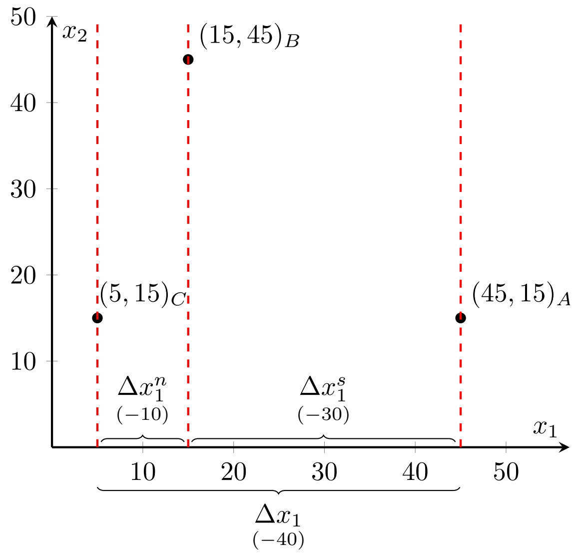
- plot the compensated budget line
- graphically identify the income and the substitution effects for good 1 and for good 2
- use an arrow to show the direction of these effects

Answer to 3.g









Question 3

- Suppose that a consumer has preferences described by the following utility function:

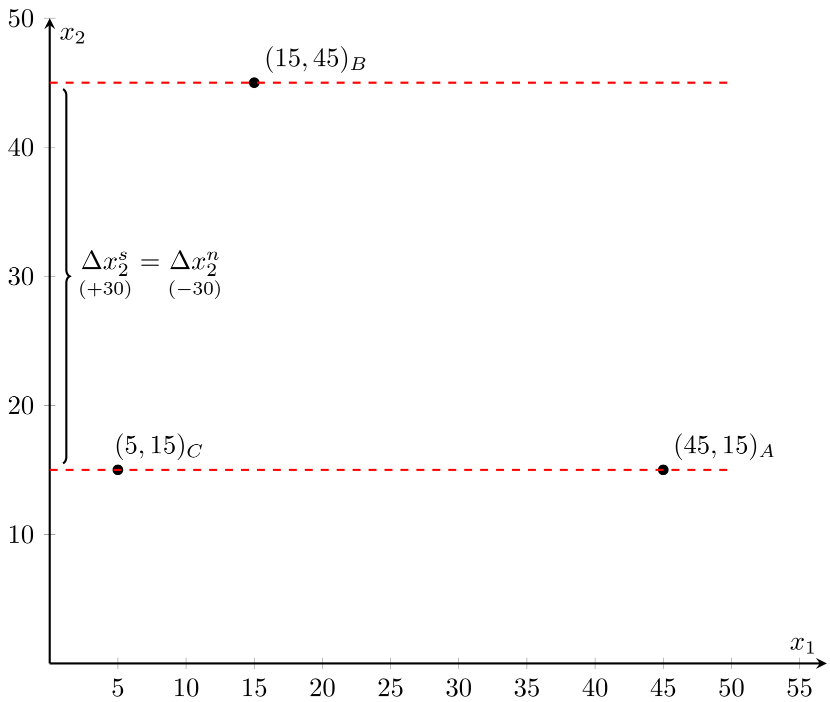
$$u(x_1, x_2) = x_1 x_2$$

- Assume that we put x_1 on the horizontal axis and x_2 on the vertical axis.
- Also, let p_1 , p_2 and I respectively denote the price of x_1, the price of x_2 and the income of the consumer.

3.h

Explain what happens to the consumption of good 2 as p_1 increases to 9.

Answer to 3.h



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